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Time-varying ambiguity, credit spreads, and the levered equity premium☆

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ABSTRACT

This paper studies the effects of time-varying Knightian uncertainty (ambiguity) on asset pricing in a Lucas exchange economy. Specifically, it considers a general equilibrium model where an ambiguity-averse agent applies a discount rate that is adjusted not only for the current magnitude of ambiguity but also for the risk associated with its future fluctuations. As such, both the ambiguity level and volatility help to raise the asset premiums and accommodate richer dynamics of asset prices. Based on a novel empirical measure of the ambiguity level, the estimated model can capture the empirical levels of corporate credit spreads and the equity premium while endogenously matching the historical default probability. More importantly, the model-implied credit spread and equity price-dividend ratio perform remarkably in tracking the time variations in their historical counterparts.

1. Introduction

As long as investors have access to multiple asset classes, prices of these assets across markets ought to equal their expected cash flows discounted by the same stochastic discount factor. Perhaps the best embodiment of this idea of consistent cross-market pricing is the structural approach to credit risk modeling, as pioneered by Black and Scholes (1973) and Merton (1974). This approach treats corporate debt and equity as contingent claims written on the same productive assets and thus builds a direct link not only between their discount-rate risks but also between their cash-flow risks. For structural credit models to be fruitfully used, especially in terms of inferring risk premiums across stock and bond markets,[1] they need to simultaneously generate equity premium and credit spreads that are consistent with the data. Empirically, however, a controversy has arisen about their ability to do so with

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[1] For example, Campello et al. (2008) and van Zundert and Driessen (2018) use corporate yield spreads to construct an ex ante measure of the equity premium, and Elkamhi and Ericsson (2008) take the opposite direction by estimating bond risk premiums using equity data.

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reasonable parameters matching historical default experience.[2]

In this paper, we show that temporal variation in ambiguity (Knightian uncertainty)[3] carries important implications for the joint valuation of debt and equity; it creates an additional uncertainty channel that helps structural models match the historical equity premium and credit spreads while endogenously generating reasonable default rates. The role of time-varying ambiguity is fulfilled through its twofold effects on (uncertainty-adjusted) discount rates.

First, consider an exchange economy where a representative agent holds multiple priors (Gilboa and Schmeidler, 1989) about aggregate consumption dynamics. To make decisions robust to model misspecifications, the ambiguity-averse agent optimally evaluates investments according to the prior that leads to the lowest expected utility. This "worst-case" evaluation brings about a first-order effect of ambiguity on discount rates, wherein the magnitude is determined by the difference between the true expected growth rate and the worst-case belief used to price assets. Second, if the set of priors is updated over time, the uncertainty about future ambiguity levels also permits a second-order effect on discount rates, in the sense that the magnitude of this effect is bound to the second moment (volatility) of the ambiguity process. In the model proposed in this paper, the presence of asymmetry in ambiguity volatility,[4] in conjunction with the agent's preference for early resolution of uncertainty, implies that the second-order effect works in the same direction as the first-order effect: a rise in the degree of ambiguity increases discount rates. These two effects jointly create a strong, negative correlation between the pricing kernel and asset valuation ratios, which per se accounts for a sizable unlevered equity premium.

However, the most interesting implication of the uncertainty-adjusted discount rates arises from endogenous default decisions. Specifically, an (positive) ambiguity shock would lower the present value of a given path of future cash flows, and thus leans shareholders toward exercising their options to default, even if there is no news of the firm's fundamentals. Therefore, time-varying ambiguity introduces a novel linkage from discount rates to corporate default timing. On the one hand, it exposes investors to great default risk exactly in the periods associated with high market prices of risk[5]; thus, high yield spreads must be demanded for investing in corporate bonds. On the other hand, equity holders are more likely to lose their cash flow rights and bear the deadweight losses exactly when their marginal utility is high. This risk channel embeds an additional component in the uncertainty premium on levered equity, making it greater than that on the unlevered asset.

Few examples of general equilibrium models succeed in explaining the equity premium and credit spreads in a unified framework: Chen et al. (2009, CCG hereinafter) build on the habit-formation model of Campbell and Cochrane (1999); Bhamra et al. (2010, BKS hereinafter), and Chen (2010) use a theoretical framework in the spirit of Bansal and Yaron (2004) that incorporates regime switching.[6] This paper departs from these existent studies in three important aspects.

1. While these studies aim to explain the equity premium and credit spreads with macroeconomic risk,[7] our explanation is based on time-varying Sharpe ratios that are driven by changes in the ambiguity level. This key difference gives our model two important advantages in fitting not only the average level but also the time series of historical credit spreads. First, it entitles our model to accounting for variations in asset premiums above and beyond what are captured by the business cycle. The economic intuition is that an ambiguity shock can lead to a large reaction in the marginal rate of substitution, even without news about the economic

[2] The key to this controversy is the definition of "reasonable parameter values." Huang and Huang (2012) find that various structural models, once calibrated to match historical default and recovery rates as well as the equity premium, underpredict corporate yield spreads (especially for investment-grade bonds). This finding is referred to as the "credit spread puzzle." Taking the view that the level of yield spreads due to non-credit factors is of similar magnitude for Aaa and Baa bonds, other studies (Chen et al., 2009; McQuade, 2016; Du et al., 2018) show a similar puzzle in Baa-Aaa yield spreads. On the other hand, David (2008) and Bhamra et al. (2010) study the impact of convexity bias on the calculation of modeled credit spreads, suggesting that the credit spread puzzle may not be as severe as documented by Huang and Huang (2012). Furthermore, Feldhütter and Schaefer (2018) criticize the practice of calibrating structural models to historical default frequencies by rating and maturity groups; they propose a new calibration approach to make the Black-Cox model generate investment-grade credit spreads that are consistent with the empirical counterparts.

[3] Ambiguity refers to the situation in which the decision-maker is uncertain about the probability law governing the state process. For notional convenience, in this paper we refer to uncertainty as both risk and ambiguity, unless Knightian uncertainty, model uncertainty, or subjective uncertainty is otherwise used in that context.

[4] Asymmetric volatility refers to the fact that an increase in volatility follows a previous rise in the ambiguity level.

[5] In addition, given that corporate defaults tend to cluster when the economic outlook becomes unclear, the liquidation process can be particularly costly during such times. Empirically, we also find that high costs of default are associated with high ambiguity levels. Consequently, the pricing kernel shows an even stronger (positive) covariance with expected default losses compared to the case in which it only comoves with the default probability, further magnifying the model-implied credit spreads.

[6] Christoffersen et al. (2017) combine two risk channels, habit formation and rare disaster, to capture the empirical level and volatility of credit spreads, as well as stylized facts in the option market. However, they embed a reduced-form model (rather than a structural one) of credit risk inside their representative-agent consumption-based model. Other studies attempting to reconcile the credit spread puzzle include David (2008), Du et al. (2018), McQuade (2016), Albagli et al. (2013), and Chen et al. (2018). These models either are not set in general equilibrium or do not consider the equity premium.

[7] As highlighted in CCG (2009), the key ingredient in all three models is the market Sharpe ratio that varies with macroeconomic conditions. However, Huang and Huang (2012) advise caution when linking the credit spread puzzle to recession risk since "there is no clear evidence yet that corporate bond defaults, especially on investment-grade bonds, are strongly correlated with business cycles." The reason is that historical variation in the aggregate default rate is almost completely driven by speculative-grade issuers, whereas the credit spread puzzle mainly concerns investment-grade bonds. As a result, countercyclical default rates do not lend empirical validity to macro-risk-based explanations for the credit spread puzzle. In other words, one needs to assess the model's explanatory power for the time variation of credit spreads, rather than that of default rates, when trying to validate the proposed risk channels.

fundamentals. Second, it allows for time variations in both the market prices of uncertainty (the first-order effect) and the quantities of uncertainty (the second-order effect).[8] The resultant flexibility explains why our model outperforms existing ones in its time-series predictions.

2. Unlike these previous studies, which focus exclusively on investment-grade issuers as inspired by Huang and Huang (2012)'s findings, we examine the credit spreads of high-yield bonds as well. The latter is equally important in examining the credit spread controversy because hypothetically we could reproduce the observed level of investment-grade spreads by imposing extremely high Sharpe ratios. But if the resulting model were to overpredict speculative-grade yield spreads, then it would merely create a "credit spread puzzle" in the other direction. With a reasonable firm-level calibration, our model can match empirical credit spreads across all rating classes.
3. Beyond the momentous aspects of asset prices, this paper also extensively studies their dynamic properties, which have not drawn sufficient attention in existing works on the credit spread controversy. Specifically, our model is able to reproduce the following empirical regularities: the procyclical variation of price-dividend ratios, the countercyclical variation of credit spreads, the long-horizon predictability of excess equity returns and credit spreads, and their weak correlations with macroeconomic fundamentals. These stylized facts are indeed reflections of the same economic mechanism, which is at the core of the model: time variation in the pricing kernel is directly linked with the cash flow risks of corporate securities through time-varying ambiguity.

With respect to implementation, an important advantage of our model is that the key driving variable, the level of ambiguity, is measurable. This facilitates our data-driven estimation of model parameters that does not involve market data. Specifically, we construct a novel measure of the economy-wide level of ambiguity by using survey forecasts. Consistent with our model's implication, higher ambiguity levels forecast higher market premiums on a broad set of assets, including equities, corporate bonds, and Treasury bonds. This ambiguity measure also exhibits a positive correlation with the historical corporate default rate, consistent with the model's prediction. To further test the empirical plausibility of the proposed economic mechanism, we estimate the default boundary with a sample of defaulting bond issuers and find that the ambiguity measure is highly significant in explaining the estimated boundary (with the expected sign). This finding substantiates the key theoretical implication that ambiguity affects the default probability by moving the default barrier.

This paper also contributes to a growing body of literature that studies representative-agent asset pricing in the presence of Knightian uncertainty. Our modeling of ambiguity aversion builds on recursive multiple-priors utility introduced and axiomatized by Epstein and Wang (1994), Chen and Epstein (2002), and Epstein and Schneider (2003). We demonstrate that our specification of time-varying ambiguity can be interpreted as a reduced form of models of learning under ambiguity, as developed in Epstein and Schneider (2007, 2008) and Illeditsch (2011). While these studies focus on model implications for the equity market, the multiple-priors preferences are also applied to other asset classes (Gagliardini et al., 2009; Ilut, 2012). However, the linkage of ambiguity aversion to the credit spread controversy has not yet been examined in the literature.[9]

Outside the multiple-priors framework, a few theoretical works also accommodate time-varying ambiguity; these studies employ a different approach to modeling ambiguity aversion—multiplier (robust-control-inspired) utility (Hansen and Sargent, 2001; Anderson et al., 2003), which defines the size of ambiguity in terms of relative entropy.[10] Maenhout (2004) proposes an ambiguity-based extension of Merton (1969, 1971, 1973)'s optimality and equilibrium theories and allows the tolerated entropy deviation to vary with the agent's wealth. In their decomposition of asset premiums into risk and (Knightian) uncertainty components, Anderson et al. (2009) break the tight link between ambiguity and wealth by letting the former depend on other state variables as well. Targeting anomalies in the option market, the model by Drechsler (2013) is most closely related to ours in the sense that the ambiguity level is also explicitly modeled as a separate stochastic process. To fully capture properties of equity and option prices, as well as the variance premium, he allows for "model uncertainty over a richer set of economic dynamics than have been possible in previous applications of robust control."[11] Consequently, ambiguity operates through multiple channels, and it is difficult to disentangle the importance of one channel from that of another. In contrast, our parsimonious model focuses on one channel—Knightian

[8] In contrast, while models of CCG (2009), Chen (2010), and BKS (2010) imply time-varying premiums as well, either the market prices of risks or the quantities of risks are held constant in their models. Le and Singleton (2010, 2013) firstly make this observation and point out the limitations resulting from these specifications.

[9] Using observations of credit default swap (CDS) spreads on financial institutions, Boyarchenko (2012) estimates a robust control model to back out the implied amount of ambiguity during the subprime mortgage crisis. Since her model does not adopt the structural approach to modeling credit risk and is not calibrated to historical default rates, it does not carry implications for the credit spread controversy.

[10] Epstein and Schneider (2003) draw a head-to-head comparison between the robust control and recursive multiple-priors models. What most distinguishes these modeling approaches is the updating rules imposed for the set of priors. To ensure dynamic consistency, multiplier preferences need a form of axiom that differs from the rectangularity employed by multiple-priors models. This form of axiom is provided by Maccheroni et al. (2006). Besides the multiple priors and multiplier utility, there is a third type of preference model that describes ambiguity-averse behavior called "smooth ambiguity" preferences (Klibanoff et al., 2005; 2009). Epstein and Schneider (2010) provide a comprehensive review of the three alternative frameworks.

[11] Drechsler (2013)'s model features a predictable (long-run) component and stochastic volatility in consumption growth, and he incorporates large jumps in these two driving state processes. The agent in his model is concerned about model misspecification with respect to all four components.

uncertainty about expected consumption growth[12]—which is absent in Drechsler (2013)'s model.

The remainder of our paper is organized as follows. In Section 2, we introduce the model and characterize the valuation of different assets in equilibrium. Section 3 describes how we measure economy-level ambiguity and illustrates the empirical relevance of the proposed ambiguity measure. Section 4 outlines the model estimation and discusses quantitative implications on asset pricing puzzles. Section 5 presents our conclusions.

2. Model framework

In this section, we build a general equilibrium model where the representative investor is uncertain about the correct probability laws governing the state process. We then allow Knightian uncertainty to vary in intensity over time and derive relevant testable implications on equity premium and credit spreads.

2.1. Modeling ambiguity aversion

Consider a measurable state space $(\Omega, \mathscr{F})$ where each $\mathscr{F}_t \in \mathscr{F}$ can be identified with a partition of Ω. Given a probability measure P, $\mathscr{F}_t$ is the σ-field generated by a d-dimensional Brownian motion B_t defined on $(\Omega, \mathscr{F}, P)$. Suppose that the representative decision-maker does not know the true probability measure P_0 and ranks uncertain consumption streams $C = \{C_t\}_{t=0}^{\infty}$, where C_t: $\Omega \to \mathscr{R}$ is $\mathscr{F}_t$-measurable. To model preferences in the presence of uncertainty, we adopt the structure of recursive multiple priors:

$$V_t^* = \min_{P \in \mathscr{P}^\Theta} E_P\left[\int_t^\infty f(C_s, V_s^P)ds \mid \mathscr{F}_t\right], \tag{1}$$

where the set $\mathscr{P}$ of priors on $(\Omega, \mathscr{F})$ is constructed by means of Girsanov transformation. In particular, we can define the Radon–Nikodym derivative (Z) of P with respect to the reference measure P_0 through a density generator (ϑ_t):[13]

$$dZ_t^\vartheta = -Z_t^\vartheta \vartheta_t dB_t, \qquad Z_0^\vartheta = 1 \tag{2}$$

$$Z_t^\vartheta = \exp\left\{-\frac{1}{2}\int_0^t |\vartheta_s|^2 ds - \int_0^t \vartheta_s dB_t\right\}, \tag{3}$$

where B_t is a Brownian motion under P_0. It follows that the generated measure $P^\vartheta(\omega) = Z(\omega)P_0(\omega)$ is equivalent to P_0. This recursive multiple-priors model (1)–(3) is proposed and axiomatized by Chen and Epstein (2002), who prove the existence and uniqueness of the solution to Eq. (1).[14]

In a pure diffusion environment, ambiguity concerns whether B_t is a Brownian motion.[15] In other words, a change of measure from P_0 to any $P \in \mathscr{P}$ affects only the drift function of the utility continuation process. To be more precise, the martingale representation theorem implies that the utility process under P_0 can be written as the solution to the backward stochastic differential equation (Duffie and Epstein, 1992):

$$dV_t^{P_0} = -f(C_t, V_t^{P_0})dt + \sigma_{v,t}dB_t. \tag{4}$$

With the multiple-priors utility, the decision-maker's acts reflect her worst-case belief. Employing the Girsanov theorem, Chen and Epstein (2002) prove that $B_t^* = \int_0^t \vartheta_s^* ds + B_t$ is a Brownian motion under the worst-case measure, where $\vartheta_s^* = \max_{\vartheta \in \Theta} \vartheta_s$.

Under Duffie and Epstein (1992)'s parameterizations of recursive preference, the aggregator function f takes the form

$$f(C, V) = \beta\theta V\left[\left(\frac{C}{((1-\gamma)V)^{\frac{1}{1-\gamma}}}\right)^{1-\frac{1}{\psi}} - 1\right], \tag{5}$$

where $\beta > 0$ is the rate of time preference, $\gamma \neq 1$ is the coefficient of relative risk aversion, and $\psi \neq 1$ is the elasticity of intertemporal substitution (EIS).[16] It follows that the stochastic control problem is to find an optimal consumption strategy c^* to maximize Eq. (1):

$$J_t = \max_{C \in \mathcal{A}} \min_{P \in \mathscr{P}^\Theta} E_P\left[\int_t^\infty f(C_s, V_s^P)ds \mid \mathscr{F}_t\right], \tag{6}$$

where $C \in \mathcal{A}$ denotes that the control process $\{C_t\}$ is admissible. Market clearing implies that the representative agent takes up aggregate consumption (i.e., $c_t^* = C_t$).

2.2. Belief sets and ambiguity shocks

The dynamics of consumption growth are specified as a diffusion process with time-varying drift:

$$\frac{dC_t}{C_t} = \mu_{c,t}dt + \sigma_c dB_{C,t}, \tag{7}$$

where the sequence $\{\mu_{c,t}\}$ is unknown to the representative agent. The agent, who observes the realized C_t, but not its drift and diffusion terms separately, is unable to tell whether an unexpected low realization of consumption growth (or even a decline) is due to a worsening economic state or just due to bad luck. The ambiguous component in Eq. (7) is manifested in a further layer of incomplete knowledge: the agent is only informed about the limiting distribution of $\mu_{c,t}$ but holds little clue about its model specification. More specifically, the empirical distribution of $\mu_{c,t}$ is known to converge to $N(\bar{\mu}, \bar{\sigma}_\mu^2)$ and be independent of $B_{C,t}$. On the other hand, the agent cannot disentangle the true data-generating process (DGP) of $\mu_{c,t}$ from

[12] Maenhout (2004) argues that the first moments of state variables are more difficult to estimate than the second moments.

[13] We assume that the regularity conditions, as specified in Appendix D of Duffie (2001), are satisfied so that Z_t^ϑ is a martingale under P_0.

[14] This well-established framework enables us to fully exploit the analytical power afforded by the continuous time. Its discrete-time version is first put forth by Epstein and Wang (1994), and the axiomatic foundations are provided by Epstein and Schneider (2003).

[15] Liu et al. (2005) and Drechsler (2013) consider agents who exhibit ambiguity aversion toward jumps in the level or in the expected (long-run) component of consumption growth.

[16] γ and ψ jointly determine the attitude toward temporal resolution of uncertainty. Because the recursive multiple-priors utility, per se, is neutral about the timing of the resolution of uncertainty (Strzalecki, 2013), the agent's temporal attitudes can be modeled separately with Eq. (5), which is the continuous-time version of Kreps–Porteus preferences.

a large family of possible processes, whose limiting distributions are identical to that of an i.i.d. normal stochastic process with mean $\bar{\mu}$ and variance $\bar{\sigma}_{\mu}^2$, even after many observations of C_t.

As discussed by Epstein and Schneider (2007), the multiplicity of probability measures in Eq. (1) indicates that the agent "has modest (or realistic) ambitions about what she can learn." In our model setup, it captures the ambiguous component $\mu_{c,t}$, which is too poorly understood to be theorized about. In response to this model uncertainty, the agent forms a set of beliefs about expected consumption growth at time t. We parameterize this set by an interval centered around the long-run mean $\bar{\mu}$,

$$\mu_{c,t}^{\mathscr{P}} \in [\bar{\mu} - A_t, \bar{\mu} + A_t], \quad A_t \geq 0, \tag{8}$$

because to the agent, the observed consumption growth is indistinguishable from a realization of geometric Brownian motion,

$$\frac{dC_t}{C_t} = \bar{\mu}dt + \sigma_g dB_{C,t}, \tag{9}$$

where $\sigma_g = \sqrt{\bar{\sigma}_{\mu}^2 + \sigma_c^2}$. It follows that A_t in Eq. (8) captures the level of ambiguity about the expected growth rates. That is, the higher A_t is, the less confidence the agent has in her probability assessment of the growth rate and the larger the set of beliefs is. With the multiple-priors utility structure, time-invariant ambiguity ($A_t \equiv A$) points to the constancy of the priors' set, which in turn indicates the lack of learning from data. This constrained specification is termed "κ-ignorance" by Chen and Epstein (2002) and adopted by Miao (2009) and Jeong et al. (2015). Removing this constraint, this paper explicitly models the time variation in A_t to allow the set of priors to actively respond to updates of information.

There are potentially many ways to obtain time-varying ambiguity endogenously through a detailed specification of the learning process, and Internet Appendix A provides a stylized model in this spirit. Specifically, this model features intangible information with ambiguous quality (Epstein and Schneider, 2008) and, more importantly, implies a stationary and persistent process of A_t. The intuition is that the ambiguity-averse agent reacts asymmetrically to signals: she tends to underweigh good news by regarding it as unreliable and overweigh bad news by fearing that it accurately conveys information about $\mu_{c,t}$. Consequently, the perceived ambiguity level moves slowly with signals and remains at a stable level in the long run.

With this insight, an exogenous, mean-reverting ambiguity process is specified for three reasons. First, modeling the origin of variation in ambiguity does not add significant economic insights for the purpose of our study. Second, it greatly complicates our model solution and estimation because the ambiguity process derived from the learning model in Internet Appendix A is non-affine. Third, it is unnecessary to model learning under ambiguity to assess the quantitative performance of our model, as we infer the historical dynamics of A_t from data on survey forecasts. Empirically, it is indeed highly difficult to feed the survey-based ambiguity measure into the learning model.

Specifically, we adopt the following affine square-root process to describe the evolution of the ambiguity level:

$$dA_t = \kappa(\bar{A} - A_t)dt + \sigma_a\sqrt{A_t}dB_{A,t}, \quad \kappa > 0. \tag{10}$$

The use of this Cox–Ingersoll–Ross (CIR) specification is an important innovation of this paper. First of all, it effectively guards against the probability that A_t falls below zero. More importantly, both the learning model in Internet Appendix A and our empirical measure in Section 3.1 suggest that A_t exhibits heteroskedasticity and asymmetric volatility; that is, as the agent becomes less confident in probability assessments, the confidence level tends to be subject to progressively larger shocks. The diffusion term in Eq. (10) captures these properties in a most parsimonious way.

2.3. Equilibrium prices

In this section, we solve for the value function J by expanding the ϑ^*-expectation of future continuation utility,

$$dV_t^{P^*} = \left[-f(C_t, V_t^{P^*}) + \vartheta_t^*\sigma_{v,t}\right]dt + \sigma_{v,t}dB_t.$$

Given that the value function is increasing in aggregate consumption, which will be verified in the following proposition, a particular form of the consumption drift—$\forall t > 0, \mu_{c,t} = \bar{\mu} - A_t$—supports the optimal choice in Eq. (1) after every history. It follows that worst-case belief P^* corresponds to the density generated by $\vartheta^* = A_t/\sigma_c$,[17] and the stochastic control problem becomes standard.

Proposition 1. With consumption and ambiguity dynamics as specified above, if $L(A_t)$ solves the following equation

$$(1-\gamma)\left(\bar{\mu} - A - \frac{1}{2}\gamma\sigma_c^2\right) + \frac{\mathfrak{D}^A L^\theta}{L^\theta} + \frac{\theta}{L} - \beta\theta = 0, \quad \gamma, \psi \neq 1, \tag{11}$$

where $\theta = (1-\gamma)/(1-\frac{1}{\psi})$ and $\mathfrak{D}^x$ is the standard Dynkin operator and it satisfies the transversality condition, then the value function is given by

$$J(C_t, A_t) = \frac{C_t^{(1-\gamma)}}{1-\gamma}(\beta L(A_t))^\theta, \tag{12}$$

and L is the price-consumption ratio in equilibrium.

[17] In our model, the exogenous state of the economy is summarized by the pair (C_t, A_t). Therefore, the density generators are technically two-dimensional, that is, $\vartheta_t^* = (A_t/\sigma_c, 0)$. This specification indicates that the agent feels uncertain only about the consumption dynamics and not about the ambiguity itself. The implicit assumption is that dispersion of professional forecasts is considered to be plausibly reflecting uncertainty about what the right model of the futures is, and the representative agent infers the dynamics of ambiguity from measures of forecast dispersion, as we will do in Section 3.1 from the perspective of econometricians. For example, suppose that in the current period, experts, whose opinions are sampled and aggregated by the agent to form her own belief set, are in closer agreement about the future path of consumption growth than they were in the last period. Consequently, the agent moves closer toward thinking in terms of a single probability measure and knows that the increased confidence in probability assessments is triggered by lower forecast dispersion. Drechsler (2013) considers an extended specification in which there is Knightian uncertainty about the dynamics of the size of ambiguous beliefs.

Comparing the Hamilton–Jacobi–Bellman equation in Proposition 1 to that with traditional expected utility, we confirm that it is unnecessary to learn what the true $\mu_{c,t}$ is to derive the model's implications. All that matters is the worst-case belief under which the agent's decisions are computed. Since Eq. (11) does not have a closed-form solution, we follow Benzoni et al. (2005) and CCG (2009) by approximating L as an exponential affine function:

$$L(A) \approx e^{\eta_0+\eta_1 A}. \tag{13}$$

Appendix B shows that this approach provides an accurate approximation to the problem solution. It also shows that the price-consumption loading on ambiguity, η_1, is negative as long as the EIS is larger than one. This corresponds to the scenario wherein the agent recoups her investment in response to shocks that blur economic prospects. Consequently, asset prices tend to drop during times of high subjective uncertainty.

In equilibrium, state prices are shaped by the agent's marginal rate of substitution. In particular, given the max-min representation (6), state prices are based on the worst-case density Z^{ϑ^*}. The following proposition sheds light on how ambiguity aversion contributes to the asset premiums.

Proposition 2. The real pricing kernel has dynamics

$$\frac{dM_t}{M_t} = -r_t dt - \Lambda_t' dB_t$$
$$-r_t dt - \left(\gamma\sigma_c + \frac{A_t}{\sigma_c}, (1-\theta)\eta_1\sigma_a A_t\right)\begin{pmatrix} dB_{C,t} \\ dB_{A,t}\end{pmatrix}, \tag{14}$$

where $r_t = \varrho_0 + \varrho_1 A_t$. The expressions ϱ_0 and ϱ_1 are given in Appendix A.

In the current setup, a nondegenerate set of priors reflects the agent's lack of confidence in her assessment of economic growth. With this interpretation, a wider span of the set demands a proportional increase in the ambiguity premium on the consumption claim, given that the lack of confidence makes the agent evaluate the asset as if the aggregate consumption grows at a rate of $\bar{\mu} - A_t$. This first-order effect is captured by the A_t/σ_c term.

In addition, the separation between risk aversion and the EIS allows the risk from fluctuations in the future ambiguity level to earn a separate premium, which is positive if the agent prefers an earlier resolution of this uncertainty ($\gamma > 1/\psi$). While this intertemporal substitution effect is defined through the second moment of ambiguity, the square-root specification in Eq. (10) makes this effect stronger when the ambiguity level is high. Hence, depending on the persistence of the ambiguity process, shocks to A_t could cause large reactions in the marginal rate of substitution. This second-order effect is characterized by the $(1-\theta)\eta_1\sigma_a A_t$ term in Eq. (14).

To summarize, the time-varying ambiguity in our model influences the market price of uncertainty in two ways. In contrast, the κ-ignorance specification would make the first-order effect degenerate into a fixed level and exclude the second-order effect. Hence, it does not allow for time-varying Sharpe ratios. While many pricing kernels engineered to explain the equity premium puzzle are able to generate time-varying asset premiums, our model is the first, to our knowledge, to permit time variations in both the market prices of uncertainty and the quantities of uncertainty.

2.4. Endogenous default and corporate security pricing

To price securities issued by individual firms, we need to specify the dynamics of corporate earnings growth, which is commonly assumed to be subject to systematic and idiosyncratic shocks in the spirit of CAPM (BKS, 2010; Chen, 2010). For tractability, we adopt two assumptions made in CCG (2009). First, the systematic component is directly tied to the growth in aggregate output O_t such that the cash flows to firm j follow the process

$$\frac{d\delta_{j,t}}{\delta_{j,t}} = \frac{dO_t}{O_t} + \sigma_j dB_{j,t}, \tag{15}$$

where $B_{j,t}$ captures the firm-specific risks, and σ_j is the idiosyncratic volatility. Second, O_t has the same growth rate as aggregate consumption C_t but with different, albeit closely correlated, dynamics:

$$\frac{dO_t}{O_t} = \mu_{c,t}dt + \sigma_o\left(\sigma_{oc}dB_{C,t} + \sqrt{1-\sigma_{oc}^2}dB_{O,t}\right), \tag{16}$$

where $dB_{O,t}$ is not correlated with either $dB_{C,t}$ or $dB_{A,t}$. That is, we interpret the claim to output as a nonlevered security such that it has a growth rate equal to that of consumption. Suppose that the agent knows the structure of earnings dynamics as presented in Eqs. (15) and (16). It follows that her subjective uncertainty about expected consumption growth would be translated into uncertainty about expected output growth.

Our benchmark model proposes that bankruptcy costs and differential tax treatment are the major market imperfections that affect corporate decisions. For tractability, we assume a stationary debt structure where a firm continuously retires a constant fraction m of existing debt and replaces it with the same amount (of principal) of newly issued debt. In making this assumption, we follow the modeling approach of Leland (1994a, 1998), who shows that although no explicit maturity is stated for the debt, the average maturity equals $1/m$. The corporate debt outstanding is composed of coupon bonds with a total coupon payment equal to C. At any time t, therefore, new bonds are issued at a rate $f = mF$, where F is the total face value of all outstanding bonds, with the instantaneous coupon rate $c = mC$ to preserve the debt structure as time elapses. On the one hand, the stockholders will have to make payments to the firm, if necessary, to cover the interest payments.[18] On the other hand, they have the contractual right to declare default at any time and turn the firm over to the bondholders. Upon default, the firm incurs a total deadweight cost equal to $\phi(A_t)U^*$, where $0 < \phi < 1$ and U^* is the unlevered asset value at the time of default.

[18] We assume that the bond indenture provisions prohibit equityholders from selling assets to pay any dividends and maintain absolute priority for bondholders.

Finally, we assume a differential tax system: corporate earnings are taxed instantaneously at a constant rate of τ_c; taxes has no loss-offset provision. Individual investors, however, pay taxes on interest income at rate τ_i and on dividend income at rate τ_d, but they are not taxed on their capital gains. It follows that the cash flows to the equity and debt issued at time t_0, when the firm is solvent, are given by

$$\delta_{e,t} = (1-\tau_e)(\delta_t - C) - mF + D(\delta_t, A_t), \quad \text{and}$$
$$\delta_{d,t} = e^{-m(t-t_0)}((1-\tau_i)C + mF),$$

respectively, where $\tau_e = 1-(1-\tau_c)(1-\tau_d)$ is the effective tax rate and D the market price of newly issued debt.

Given that it is in the interest of the equityholders to choose when to default in such a way that the value of equity is maximized, the endogenous default can be formulated as an optimal stopping problem. Fix a domain $\mathcal{S} \subset \mathbb{R} \times (0, \infty)$ for the state vector (δ_t, A_t), and define $\tau_{\mathcal{S}} = \inf\{t > 0; (\delta_t, A_t) \notin \mathcal{S}\}$. Then the optimal default boundary is determined by finding a stopping time $\tau^*(\delta_t, A_t)$ such that

$$E(\delta_t, A_t) = E_t^Q\left[\int_t^{\tau*} e^{-\int_t^s r_u du}\delta_{e,s} ds\right] = \sup_{\tau \in \mathcal{T}} E_t^Q\left[\int_t^{\tau} e^{-\int_t^s r_u du}\delta_{e,s} ds\right], \tag{17}$$

where $\mathcal{T}$ is the set of all stopping times $\tau < \tau_{\mathcal{S}}$. Eq. (17) formulates an optimal stopping problem in which $\delta_{e,t}$ is the “utility rate” function, and the “bequest” function is zero (since equityholders receive nothing at default). It can be linked to the free boundary problem through the high contact principle (Øksendal, 1990). Given this insight, Appendix C presents a partial differential equation (PDE) characterization of the equity valuation. To solve these PDEs, we perform a regular perturbation analysis based on the time-series properties of the ambiguity process.

As discussed in Section 2.2, the agent shapes her set of priors by sampling and aggregating experts' opinions such that she can measure the level of Knightian uncertainty in each period and infer its law of motion. We assume that A_t is perceived to follow a persistent process, of which the theoretical foundation is laid out by the learning model in Internet Appendix A.[19] We capitalize on this persistence by adding a small time-scale parameter ε to its dynamics:[20]

$$dA_t = \varepsilon\kappa_A(\bar{A} - A_t)dt + \sigma_a\sqrt{\varepsilon A_t}dB_{A,t}. \tag{18}$$

The small magnitude of ε captures the slow mean-reversion in A_t and makes possible the following expansion with respect to equity prices:

$$E = E_0 + \sqrt{\varepsilon}E_1 + \varepsilon E_2 + O(\varepsilon^{\frac{3}{2}}), \tag{19}$$

where the expansion series solve a series of ODEs, as presented in Internet Appendix D.

To obtain economic insights from this approximate analytic solution, we focus on the leading-order terms, which essentially solve a reduced, one-dimensional free boundary problem. The expression for E_0 in Eq. (19) is similar to Leland (1994a)'s pricing formula with ambiguity fixed at the current level

$$E_0 = (1-\tau_e)\delta L^o(A) + (\tau_e - \tau_i)C\mathscr{PV}^r(A)\left(1 - \left(\frac{\delta}{\delta_0^*(A)}\right)^{\alpha_r}\right) - \phi\delta_0^*(A)L^o(A)\left(\frac{\delta}{\delta_0^*(A)}\right)^{\alpha_r} - \frac{D_0(\delta, A)}{m}, \tag{20}$$

where $L^o(A)$ denotes the ratio of a debtless firm's value to its earnings, and $\mathscr{PV}^r(A)$ the perpetuity factor; both terms will be approximated as an exponential linear function of the ambiguity level in Appendix C. The expressions for α_{r+m}, α_r, and D_0 are given in Internet Appendix D. δ_0^* is the leading-order term in an according asymptotic expansion of the optimal default boundary:

$$\delta^* = \delta_0^* + \sqrt{\varepsilon}\delta_1^* + \varepsilon\delta_2^* + O(\varepsilon^{\frac{3}{2}}), \tag{21}$$

and it has the following solution

$$\delta_0^* = \frac{((1-\tau_i)C + mF)\alpha_{r+m}\mathscr{PV}^{r+m}(A) - (\tau_e - \tau_i)C\alpha_r\mathscr{PV}^r(A)}{(\phi\alpha_r + (1-\phi)\alpha_{r+m} - 1)L^o(A)}. \tag{22}$$

As indicated by Eqs. (20) and (22), the leading-order terms are equivalent to an application to credit risk modeling of the κ-ignorance specification, where the degree of ambiguity is constant over time. Intuitively, the persistence in the A_t process makes the agent attach great importance to its current level rather than its long-run mean; as such, $\bar{A}$ does not appear in the primary-order terms.

For this reason, the agent's valuation of unlevered earnings claims shows large negative reactions to ambiguity shocks, which are perceived as being long-lasting. As implied by Eq. (22), this strong and negative correlation between the price-earnings ratio $L^o(A)$ and the ambiguity level makes δ_0^* an increasing function of A. It follows that equityholders prefer earlier default when growth prospects become highly uncertain. Combining this result with the positive correlation between ambiguity and the stochastic discount factor (SDF), as implied by Proposition 2, we obtain that default events tend to cluster exactly when the marginal utility is high. Hence, the agent demands a higher premium on corporate bonds than would be the case if there were a constant default barrier.

This high ambiguity elasticity of unlevered asset prices not only enables the model to generate sizable credit spreads but also helps to explain the equity premium puzzle. The first term on the right-hand side of Eq. (20) represents the after-tax value of unlevered assets. Again, given the negative reaction of the price-earnings ratio to ambiguity shocks, we can obtain a negative covariance between the pricing kernel and the price of the unlevered

[19] The A_t derived from this learning-under-ambiguity model moves slowly with signals, of which the precision and relevance are uncertain. The simulation analysis based on the calibrated learning model indicates that the mean of first-order autocorrelations across trials is 0.832.

[20] If A_t turned out to be a fast-moving process, we can replace ε with $1/\varepsilon$, where ε is still a small parameter. Notably, this numerical approach is also used by McQuade (2016), who proposes a resolution of the credit spread puzzle as well, by incorporating stochastic volatility into structural modeling. Yet in contrast to McQuade (2016), who draws a distinction between growth and value firms and aims to provide cross-sectional implications for market and book values of firms' equity, we attempt to derive the effects of time-varying ambiguity on the prices of various asset classes in a general equilibrium model.

equity, which is the result researchers commonly pursue when attempting to resolve the equity premium puzzle. On the other hand, our model introduces an additional component in levered equity premium, as captured by the second term: the (present value) of tax benefits. Since tax benefits cannot be claimed after default, their value equals the product of the tax-sheltering value of coupon payments ($(\tau_e - \tau_i)C$) and the probability of solvency $(1 - (\delta/\delta_0^*)^{\alpha_r})$; the latter is directly determined by the default threshold $\delta_0^*(A)$, whose relation with the ambiguity level is shown in Eq. (22). It follows that equityholders are more likely to lose the tax shelter when their marginal utility is high. Therefore, the model-implied levered equity premium is higher than its unlevered counterpart.[21]

3. Empirical evidence

To test the main implications of our model, we create a proxy for economy-wide Knightian uncertainty. It is used in this section to provide validation for the model-suggested effect of time-varying ambiguity on asset premiums. As will be shown in the next section, it also greatly facilitates our model estimation and identification of the model-implied time series of equity prices and credit spreads.

3.1. Measuring the level of Knightian uncertainty

In our model, the level of ambiguity is captured by the "distance" between the most optimistic and pessimistic outlooks on economic growth. Accordingly, our empirical proxy $\widetilde{A}_t$ is constructed as the cross-sectional range of professional forecasts of next quarter's real output growth; it clearly maps to A_t in our model:

$$\widetilde{A}_t = \Gamma A_t, \quad \text{where } \Gamma = \frac{2\sigma_o \sigma_{oc}}{\sigma_c}.$$

The underlying assumption is that the representative agent aggregates and synthesizes survey forecasts to form her own belief set. Consequently, the more widely dispersed opinions are from survey participants, the lower confidence she has in probability assessments of the future.

The data source used in this study is Blue Chip Financial Forecasts (BCFF), which conducts monthly surveys that ask approximately 45 financial market professionals for their projections of a set of economic fundamentals covering real, nominal, and monetary variables.[22] To prevent the forecast horizon from varying over time,[23] we sample individual forecasts at a quarterly frequency such that the horizon is fixed at three months. Dictated by the availability of forecasts of real GDP (GNP) growth, the sample period for the rest of this article is from 1985Q1 to 2010Q4.

In practice, the range of survey forecasts, like other nonrobust statistics, could be unduly affected by outlier responses. For example, since 2002 Genetski.com has consistently made predictions of GDP growth that are about 1% higher than any other respondents, until it stopped participating in the survey in October 2004. Including this single data point would increase our ambiguity measure by at least 1%. To minimize the impact of such outliers, we employ in our analysis the interval between the 90th percentile point and the 10th percentile point of each cross-section,[24]

$$\widetilde{A}_t = \hat{F}_t^{-1}(0.9) - \hat{F}_t^{-1}(0.1),$$

where $\hat{F}_t(x)$ denotes the time-t empirical distribution of individual forecasts. The use of this interdecile range is conceptually consistent with the multiple priors utility; it is unlikely that the agent simply pools experts' opinions without any screening when developing her set of priors. As discussed in Gajdos et al. (2008), the subjective belief set should be distinguished from the set of all logically possible priors, which contains those outliers.

It is important to note that our range-based measure is motivated by the rectangularity of the belief set,[25] a key requirement in multiple-priors models for dynamic consistency. Accordingly, studies based on multiplier preferences tend to measure ambiguity by the cross-sectional variance of survey forecasts, which is consistent with their entropy-based formulation of the agent's belief set. For example, Anderson et al. (2009, AGJ hereinafter) construct an index of Knightian uncertainty as a weighted variance of forecasters' predictions of the (excess) market return, which are imputed from their predictions of several related economic variables; Drechsler (2013) takes the standard deviation in the output forecasts as a proxy for the economy-wide level of ambiguity. In Appendix A.1, we

[21] The third term in Eq. (20) derives from the reorganization costs upon bankruptcy; it is the product of three components: 1) the fractional bankruptcy cost ϕ, 2) the value-based default boundary $\delta_0^* L^o$, and 3) the default probability $(\delta/\delta_0^*)^{\alpha_r}$. As will be shown in Section 4, ϕ and $(\delta/\delta_0^*)^{\alpha_r}$ are positively dependent on the ambiguity level, but $\delta_0^* L^o$ is negatively dependent. So the effect of ambiguity on the expected default cost can only be examined numerically. Given the estimated and calibrated parameters in Section 4, we find that for Baa firms, the expected deadweight loss is a nondecreasing function of A (at least over its 90% interval based on the stationary distribution). Therefore, shareholders are supposedly prepared for high default costs when the ambiguity level is high.

[22] More details on this survey and its comparison to an alternative source, the Survey of Professional Forecasters (SPF), are discussed in Internet Appendix B. The advantages of BCFF over SPF are also discussed in, for example, Buraschi and Whelan (2012) and Buraschi et al. (2018). To the best of our knowledge, the current study is the first to infer the ambiguity level using the Blue Chip data. We directly measure the uncertainty about output growth, rather than consumption growth, because BCFF does not ask participants for forecasts of real consumption expenditures. While the SPF contains this category, the responses are not collected by type of product. Hence, forecasts of nondurables plus service consumption (the variable typically used for model calibration in the literature) are unavailable.

[23] Within the BCFF survey, which is published monthly, forecasts are always made for a specific calendar quarter.

[24] Before 1990, each release of the Blue Chip survey contained two statistics named "TOP10" and "BOT10," respectively. They represented not the 90th and 10th percentile points but the average values among top ten and bottom ten predictions.

[25] Ilut and Schneider (2014) also use the interdecile range of real GDP forecasts, constructed from the SPF data, to infer the confidence about TFP. In an early version of their paper (Ilut and Schneider, 2011), the interquartile range—the difference between the upper and lower quartiles—is used as the ambiguity measure.

draw a comparison of $\widetilde{A}_t$ with alternative ambiguity measures.[26]

Fig. 1 shows the dynamics of $\widetilde{A}_t$, which is plotted as the dashed blue line.[27] We find that it widely fluctuates around an average value of 2.1%, ranging from 3.8% in the early 1990s recession to about 1.5% during most years under the Clinton administration. Moreover, taken together with the business cycle, its historical variation seems too systematic to be attributed to pure sampling errors. On the other hand, the figure also reveals that as investors become less confident in their probability assessments, the confidence level tends to be subject to progressively larger shocks. We present statistical evidence in Internet Appendix B.2 for time-varying and asymmetric volatility in the evolution of $\widetilde{A}_t$. Overall, the mean-reverting, square-root process as exogenously specified in Section 2.2 aptly summarizes the key dynamic features of $\widetilde{A}_t$.

Fig. 1 provides visual evidence of the comovement of aggregate ambiguity with corporate default frequency and the price-dividend ratio on the CRSP value-weighted market index. The red line presents Moody's annual issuer-weighted default rates.[28] As noted by Chen (2010) and BKS (2010), we can observe significant countercyclical fluctuations in the default rates, which tend to rise before contractions and peak at the troughs of recessions. However, a closer inspection also reveals that the two lines closely correspond to each other even within business cycles. These remarks apply equally to the price-dividend ratio: while its procyclical variation has been documented extensively, the figure shows that it (negatively) covaries with the ambiguity measure at a frequency higher than the business cycle. Indeed, their correlation is measured at −73.5%, which is much more remarkable than the correlations of the P/D ratio with real output and consumption growth (12% and 21%). Overall, Knightian uncertainty seems to capture both the inter- and intra-cycle variations in default probabilities and stock prices.

A well-documented regularity in empirical asset pricing is that the variance of price-dividend ratios corresponds almost entirely to discount rate variation (rather than variation in expected dividend growth). Consequently, the comovement between the P/D ratio and $\widetilde{A}_t$ suggests that ambiguity about economic growth has a positive effect on the discount rate—an important proposition derived from our model. This result, in conjunction with the positive correlation between ambiguity and the default rate, constitutes a necessary condition for our model to generate sizable credit spreads.

3.2. Predictability of asset returns and credit spreads

Besides its pricing implication, our model makes further predictions about the role of ambiguity in driving time variations in the equity premium and credit spreads. In this section, we aim to examine whether the proposed ambiguity measure has explanatory power for market premiums on a number of assets. To this end, we first examine its significance as a predictor of stock returns:

$$r^M_{t+1} = \text{constant} + b\widetilde{A}_t + Z_t + \epsilon_t,$$

where r^M_{t+1} is defined as the difference between the quarterly return on the CRSP value-weighted portfolio and the corresponding three-month T-bill rate, and Z_t denotes potential control variables. Since $\widetilde{A}_t$ is constructed from three-month-ahead forecasts made at the beginning of each quarter, the horizon of survey forecasts matches that of our predictive regression. This specification maps exactly to the concept of one-step-ahead conditionals in recursive multiple priors.

Panel A of Table 1 reports the regression estimates. Since the quarterly regression involves nonoverlapping observations, t-statistics in parentheses are simply based on Newey–West standard errors adjusted for six lags of moving average residuals. The first row in Panel A shows that the ambiguity measure has significant unconditional predictive ability for excess market returns, with adjusted R^2 equal to 4.3%. Higher levels of ambiguity are associated with higher equity returns, implying that Knightian uncertainty is priced. Moreover, the magnitude of its impact is sizable as well: a one-standard-deviation (0.75%) increase in $\widetilde{A}_t$ raises the expected quarterly return by 2.18%. We also examine the (unconditional) predictive power of other ambiguity measures in Appendix A.1, and the AGJ measure turns out to be the only significant one.[29]

In remaining rows in Panel A, we control for return predictors that have been shown to hold short-horizon forecasting power in previous studies.[30] These include the

[26] It is also notable that in the literature of heterogeneous beliefs models, forecast dispersion is also used to measure agents' disagreement about economic fundamentals. The specific statistics used to construct related measures include the standard deviation (Anderson et al., 2005; Buraschi and Jiltsov, 2006) and the mean absolute deviation (Buraschi et al., 2014). The empirical properties of these difference-in-beliefs indices are examined in Appendix A.2. One may question whether $\widetilde{A}_t$ actually captures the signal-to-noise ratio and thus aggregate volatility. We assess the empirical plausibility of this interpretation in Appendix A.3. While there are a couple of studies constructing measures of macroeconomic volatility based on survey forecast data (Bansal and Shaliastovich, 2013; Le and Singleton, 2013), they focus on time-series variation in the consensus (average) forecast rather than the cross-sectional dispersion of individual forecasts. We show that $\widetilde{A}_t$ has a limited correlation with their empirical measures and with alternative volatility proxies based on realized and implied variances of market returns. In particular, its role in driving asset premiums is entirely unaffected by the inclusion of volatility measures.

[27] In Fig. 1, $\widetilde{A}_t$ is sampled annually to match the frequency of corporate default rates.

[28] They are reported in Exhibit 31 of Moody's annual default study "Corporate Default and Recovery Rates, 1920–2010".

[29] Its cross-sectional correlation with $\widetilde{A}_t$ is rather low, which is not surprising as they are designed to capture the Knightian uncertainty about different economic concepts: the conditional mean of market returns and that of output growth. To substantiate this point, we apply the statistics underlying the AGJ measure—a weighted variance with the weights mimicking a beta distribution—to forecasts of real GDP growth and find that the resultant measure shows a much stronger correlation with $\widetilde{A}_t$ (at 53.6%). This finding may reflect some common features of the weighted variance and the interdecile range; for example, both of them carry out the function of minimizing the impact of extreme forecasts.

[30] In Internet Appendix B.3, we consider other conditioning variables known to be effective in forecasting equity returns at long horizons (one year or longer). These variables include the net payout ratio (Boudoukh et al., 2007), detrended risk-free rates (Fama and Schwert, 1977), and Lettau and Ludvigson (2001)'s *cay*.

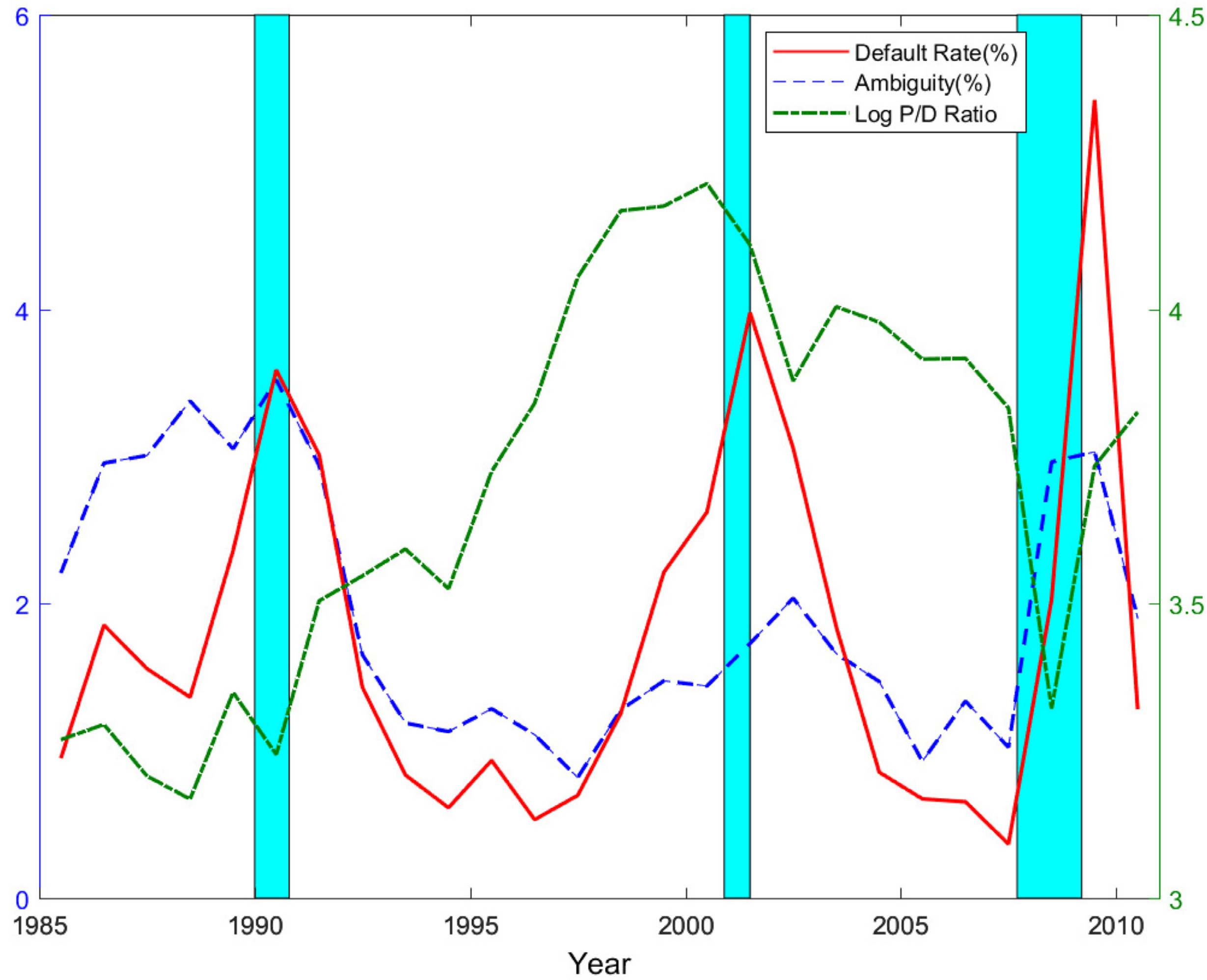

Fig. 1. Historical default rates, levels of ambiguity, and price-dividend ratios.
This figure shows the Moody's corporate default rates (solid red line), the annualized measure of ambiguity level constructed from professional forecasts (dashed blue line), and the logarithm price-dividend ratio on the NYSE/Amex/Nasdaq value-weighted index (dash-dot green line). The *y*-axis on the left side applies to the first two time series, and the *y*-axis on the right side the last. Shaded bars denote months designated as recessions by the National Bureau of Economic Research. The sample period spans from 1985 to 2010. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

output gap measured using the approach of Cooper and Priestley (2009), the ratio of new orders to shipments of durable goods (Jones and Tuzel, 2013), and the three-month growth rate of the Baltic Dry Index (Bakshi et al., 2012). We find that the economic and statistical significance of our ambiguity measure remains similar. Among the three conditioning predictors, the Baltic Dry Index accomplishes the greatest return predictability when combined with $\widetilde{A}_t$; the resultant adjusted R^2 is 11.2%.

To further pin down the role of ambiguity in pricing corporate securities, we estimate the following predictive regressions of credit spreads for various rating classes:

$$CS_{t+1} = \text{constant} + b\widetilde{A}_t + \epsilon_t,$$

where CS_{t+1} is the quarter-end yield spreads between Barclays (Lehman) US corporate bond and Treasury bond indices. Panel B of Table 1 contains the results of forecasting yield spreads of Aaa- to B-rated bonds. All regression coefficients are significant at the 5% level, and $\widetilde{A}_t$ seems to have stronger predictive power for speculative-grade bonds. The latter is not surprising, as ambiguity only affects the pure credit component in yield spreads, which accounts for a greater fraction in those lower rating categories (Huang and Huang, 2012; Avramov et al., 2007b; Rossi, 2014).

The predictive content of $\widetilde{A}_t$ is not limited to corporate securities. Panel C presents the performance of $\widetilde{A}_t$ in forecasting three-month excess returns on Treasury bond portfolios. We find that ambiguity explains a significant fraction of the variations in both intermediate-term and long-term bond returns. Again, the coefficient estimates are positive, indicating that in an ambiguous environment

Table 1
Asset return and credit spread predictability by the ambiguity level.
This table contains results from regressing (one-quarter-ahead) excess stock returns, credit spreads, and excess bond returns on the proposed ambiguity measure alone or with other predictors. Conditioning predictors include the output gap measure of Cooper and Priestley (2009), the new-orders-to-shipments ratio of Jones and Tuzel (2013), and the growth rate of the Baltic Dry Index (Bakshi et al., 2012). r_t^M is defined as the difference between the continuously compounded return on the CRSP value-weighted index and the contemporaneous return on a three-month Treasury bill. CS_t^{Aaa} - CS_t^B denote the credit spreads of Barclays (Lehman Brothers) bond indices. r_{t+1}^{Int} and r_{t+1}^{Long} are quarterly returns on the Barclays long-term and intermediate-term Treasury indices in excess of the contemporaneous return on a three-month Treasury bill. t-statistics are computed following the procedures of Newey and West (1987) with a lag truncation parameter of six. The last two columns report Campbell and Thompson (2008)'s out-of-sample R^2s as well as p-values of the Clark-West (2007) test. The benchmark model in the Clark-West test is based on the historical average return (spread) or one of conditioning predictors. The sample period spans from 1985Q1 to 2010Q4.

	In-sample					Out-of-sample	
	Ambiguity	Output gap	ln NO/S	BDI growth	Adjusted R^2	Out-of-sample R^2	Clark–West p-value
Panel A: Predictability in excess stock market returns							
r_{t+1}^M	2.923				0.043	0.038	0.035
	(2.736)						
r_{t+1}^M	2.884	−0.136			0.042	0.030	0.041
	(2.349)	(−0.761)					
r_{t+1}^M	2.947		−0.436		0.053	0.038	0.023
	(2.707)		(−1.540)				
r_{t+1}^M	3.249			0.120	0.112	0.061	0.019
	(2.568)			(3.163)			
r_{t+1}^M	2.661	−0.062	−0.368	0.119	0.120	0.039	0.047
	(2.112)	(−0.454)	(−1.495)	(3.277)			
Panel B: Predictability in corporate yield spreads							
CS_{t+1}^{Aaa}	0.239				0.093	0.072	0.007
	(2.140)						
CS_{t+1}^{Aa}	0.384				0.159	0.122	0.022
	(3.026)						
CS_{t+1}^{A}	0.473				0.144	0.130	0.006
	(2.764)						
CS_{t+1}^{Baa}	0.566				0.137	0.126	0.002
	(2.842)						
CS_{t+1}^{Ba}	1.438				0.260	0.236	0.000
	(3.014)						
CS_{t+1}^{B}	1.939				0.262	0.255	0.004
	(3.536)						
Panel C: Predictability in excess Treasury bond returns							
r_{t+1}^{Int}	1.112				0.032	0.025	0.030
	(3.091)						
r_{t+1}^{Int}	1.180	−0.004			0.027	0.022	0.026
	(2.975)	(−0.303)					
r_{t+1}^{Int}	1.167		−0.040		0.030	0.032	0.042
	(2.919)		(−1.865)				
r_{t+1}^{Int}	1.231	0.007	−0.043		0.036	0.029	0.039
	(3.024)	(0.449)	(−2.014)				
r_{t+1}^{long}	2.774				0.024	0.019	0.064
	(2.416)						
r_{t+1}^{long}	2.397	−0.000			0.019	0.019	0.053
	(2.210)	(−0.014)					
r_{t+1}^{long}	2.208		−0.061		0.021	0.023	0.061
	(2.181)		(−1.211)				
r_{t+1}^{long}	2.181	0.019	−0.070		0.023	0.026	0.066
	(2.262)	(0.561)	(−1.354)				

premiums tend to be higher.[31] Since Cooper and Priestley (2009) and Jones and Tuzel (2013) show that their predictors can forecast excess bond returns as well, we also assess the predictive power of $\widetilde{A}_t$ conditional on these factors and find that $\widetilde{A}_t$ retains its statistical significance.

To reinforce the common predictor conclusion, we perform an out-of-sample analysis based on two metrics. We employ, first, the out-of-sample R-squared proposed by Campbell and Thompson (2008) and, second, the approximately normal test statistics developed in Clark and West (2007). Both statistics are standard in the literature of return predictability (Jones and Tuzel, 2013; Bakshi et al., 2012). In keeping with these studies, we choose a ten-year initialization period to "train" predictive regression models.

[31] The model presented in Section 2 implies that the risk premiums on real bonds are low when the ambiguity level is high. If we introduce a positive correlation between innovations in ambiguity and expected inflation, the model generates a positive effect of ambiguity on nominal bond risk premiums; this positive correlation exists in the data if expected inflation is measured by consensus survey forecasts (Bansal and Shaliastovich, 2013).

When evaluating the unconditional predictive power of $\widetilde{A}_t$, we always compare it to the mean of all return/spread observations up to and including time-t.

As shown in the seventh column of Table 1, out-of-sample R^2s are uniformly positive in univariate regressions. They suggest that using $\widetilde{A}_t$ as the predictor leads to a lower mean squared forecast error. Among different asset classes, the premiums on corporate bonds appear subject to the highest predictability, as in-sample results reveal. The p-values from the Clark–West tests confirm the effectiveness of $\widetilde{A}_t$ as a real-time predictor.[32] The only exception is long-term Treasury bonds, for which the ambiguity measure is significant only if the test's size is set at 10%. Otherwise, none of the Clark–West p-values exceeds 0.04. As to multivariate regressions, we carry out Clark–West tests of the null hypothesis that regressions with and without $\widetilde{A}_t$ lead to equal forecast accuracy.[33] The test results mirror those regarding univariate regressions: $\widetilde{A}_t$ contains significant additional information in forecasting excess returns on stocks and intermediate-term bonds; in predictive regressions of long-term bond returns, its added value is significant at the 10% level. Generally, we find that $\widetilde{A}_t$'s in-sample significance is translated into the out-of-sample context.

4. Model implications

In this section, we assess the model's empirical performance in the joint valuation of debt and equity. Particularly, we set the κ-ignorance specification as the benchmark to illustrate the quantitative impact of time-varying ambiguity on asset prices. We start with a maximum likelihood estimation (MLE) of the state space. Section 4.2 considers the model's ability to account for the observed credit spreads with a reasonable firm-level calibration. In Section 4.3, we investigate whether the model can match some key moments of the levered equity premium and risk-free rate. The last subsection examines the model's implications for the historical dynamics of asset prices.

4.1. Data and estimation

A major obstacle to the estimation of equilibrium asset pricing models is that the key state variable—for example, the surplus consumption ratio or the long-run component in consumption growth—tends to be unobservable. Advantageously, this model features a fully measurable state vector. Therefore, we obtain model parameters through MLE while leaving the choices of preference parameters similar to the existing studies. Specifically, we consider the state vector $Y_t = \{c_t, o_t, \widetilde{A}_t\}$, which follows an affine process (Dai and Singleton, 2000; Duffie et al., 2003):

$$dY_t = (\mathcal{A} + \mathcal{B}Y_t)dt + \Sigma\sqrt{S_t}dB_t, \qquad \text{where} \tag{23}$$

$$\mathcal{A} = \begin{bmatrix} \bar{\mu} - \frac{1}{2}\sigma_g^2 \\ \bar{\mu} - \frac{1}{2}\sigma_o^2 \\ \kappa\Gamma\bar{A} \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 0_{3\times 2} & \begin{matrix} -\frac{\sigma_g^2\sigma_{ca}^2}{2\Gamma} \\ -\frac{\sigma_o^2\sigma_{oa}^2}{2\Gamma} \\ -\kappa \end{matrix} \end{bmatrix},$$

$$\Sigma = \begin{bmatrix} \sigma_g & 0 & \frac{\sigma_g\sigma_{ca}}{\sqrt{\Gamma}} \\ \sigma_o\sigma_{oc} & \sigma_o\sqrt{1-\sigma_{oc}^2} & \frac{\sigma_o\sigma_{oa}}{\sqrt{\Gamma}} \\ 0 & 0 & \sqrt{\Gamma}\sigma_a \end{bmatrix}, \quad B_t = \begin{bmatrix} B_{C,t} \\ B_{O,t} \\ B_{A,t} \end{bmatrix}$$

$$S_{11,t} = S_{22,t} = 1, \quad S_{33,t} = \widetilde{A}_t, \quad S_{ij,t} = 0 \text{ for } i \neq j, 1 \le i, j \le 3.$$

Table 2
Estimation of model parameters.
This table reports estimated model parameter values based on the closed-form maximum likelihood estimation of the model using US data at a quarterly frequency. The sample period spans from 1985Q1 to 2010Q4. Quantities shown in parentheses are the misspecification-robust standard errors obtained using the Huber sandwich estimator (Aït-Sahalia, 2012). They are reported as the significand sharing the same scientific notation with the corresponding point estimate.

Parameter	Estimate	s.e.	Parameter	Estimate	s.e.
Consumption & output			Ambiguity		
$\bar{\mu} \times 10^2$	1.54	(0.32)	κ	0.30	(0.10)
$\sigma_g \times 10^2$	0.82	(0.19)	$\bar{A} \times 10^2$	1.11	(0.12)
σ_{ca}	−0.11	(0.04)	$\sigma_a \times 10^2$	8.13	(2.05)
$\sigma_o \times 10^2$	1.21	(0.22)			
σ_{oc}	0.64	(0.11)			
σ_{oa}	−0.39	(0.08)			

Compared to the state process developed in Section 2, Eq. (23) allows for the correlation among innovations in the model estimation. For example, we find that, empirically, a loss of confidence (in probability assessment) is bad news for future consumption and output growth. Indeed, the impulse response function shows that a one-standard-deviation shock lowers consumption growth over the next quarter by roughly six basis points.

Following the convention in the literature, C_t is measured as the sum of real personal consumption expenditures on nondurable goods and services and O_t as real GDP per capita.[34] Both data series are retrieved from the US national accounts of the Bureau of Economic Analysis and are seasonally adjusted. The estimation frequency is quarterly, and the sample period is 1985 to 2010. It is worth emphasizing that our estimation of the state dynamics does not involve any asset prices.

Since the conditional density of Y_t cannot be extracted in closed form, we apply the approximate MLE developed by Aït-Sahalia (1999, 2002, 2008), who constructs explicit expansions for the log-likelihood function of a large class of univariate and multivariate diffusion processes.[35] The estimation results are presented in Table 2. A most

[32] As Clark and West (2007) show both theoretically and numerically, their test statistic is approximately normal after adjusting for the estimation error of the larger model. Thus, we only report the p-values.

[33] In the computation of out-of-sample R^2s, the benchmark regression is still based on the historical average of the predicted variable.

[34] They have almost identical mean growth rates (1.58% versus 1.54%) over our sample period, and the correlation between their innovations is measured at 63.1%. This evidence validates our assumption in Section 2.4: the mean of output growth is chosen to match that of consumption growth, but they are allowed to have different dynamics.

[35] Note that Y_t follows an irreducible process, as defined in Aït-Sahalia (2008). Therefore, each term in the expansion series needs to be solved from Kolmogorov forward and backward equations. To obtain an analytic expression for these terms, we follow Aït-Sahalia's procedure for approximating the likelihood function two-dimensionally: in the time interval $\Delta = 1/4$ and in the state variable $Y_{t+\Delta} - Y_t$.

remarkable aspect of parameter estimates pertains to the persistence of the ambiguity level. The mean-reversion rate is estimated at 0.30, corresponding to a mean return time of more than three years; this finding substantiates our use of the perturbation approach for security pricing under slow-variation asymptotics of the ambiguity process. The unconditional expectation of ambiguity $\bar{A}$ is about 1.11%. This quantity, when combined with estimates of σ_c, σ_o, and σ_{oc}, implies that the proposed measure $\widetilde{A}_t$ has a long-run mean of 2.09%, which matches almost exactly the average value observed from the survey data. Finally, the estimate of σ_{ca} is negative and significant at the 1% level.

The preference parameters reported in Table 2 are chosen to take into account the empirical evidence and economic considerations. The time discount factor β in the stochastic differential utility is usually calibrated at somewhere between 0.01 and 0.02.[36] Taking an intermediate value in this range, we fix β at 0.015. We further assume that the RRA coefficient γ is equal to 10, as do Bansal and Yaron (2004). The question of whether the magnitude of the EIS is greater than one is controversial. We set its value to Bansal and Shaliastovich (2013)'s estimate of 1.81, which they derive from bond market data as well as survey forecasts.[37]

4.2. The credit spread puzzle

In a step toward developing a unified understanding of how time-varying ambiguity affects the pricing of equities and corporate bonds, this section examines the model's implications for credit spreads. Using calibration methodologies commonly employed in the literature, we choose firm-level parameters to match the average solvency ratios of firms in different rating categories. However, in this calibration experiment, historical default rates are not targets toward which we calibrate the model. Instead, they serve as criteria used to assess the model performance. This procedure for generating endogenous default probabilities is consistent with BKS (2010).

4.2.1. Firm-level calibration

A key variable in the valuation of default claims is the cost of default ϕ. Given that the default timing and default boundary are endogenous in our model, the default loss, too, should be modeled to allow for covariation with the state of the economy. As will be illustrated in Section 4.2.3, our model indicates that the default timing tends to be positively correlated with the degree of ambiguity. It follows that financial distress would be enormously costly when prospects for economic growth are highly unclear (Shleifer and Vishny, 1992). Given this insight, we parameterize the default cost as a linear function of the ambiguity level and estimate relevant coefficients with a market-based approach. Specifically, we follow Davydenko et al. (2012) by collecting observations of ϕ from the market values of defaulting firms and then regress them on our ambiguity measure. Details of our data sources and empirical methodology are presented in Internet Appendix C. We obtain the following regression results:

$$\phi_{j,t} = \underset{(6.61)}{0.113} + \underset{(5.28)}{6.294} \quad \widetilde{A}_t.$$

The effect of ambiguity on the default cost is statistically significant and economically substantial. The regression estimates can be translated to a slope coefficient of 6.294 for A_t; it implies that a 1% increase in A_t pushes up the proportional cost by 11.8%.

Table 3 summarizes several calibrated firm-level parameters in addition to bankruptcy costs. Parameters on tax rates are set to be consistent with Chen (2010)'s calibration, as reported in Panel A. Panel B shows calibration targets that enable the model to generate differential default rates across credit ratings. Because our model defines default time in terms of a firm's cash flow rather than its asset value, we directly calibrate the debt structure to earnings-based financial metrics.[38] That is, we match the initial proportion of F to δ_0 to the historical debt-earnings ratio for each rating category, which is retrieved from the special reports of Moody's Financial Metrics.[39] By the same principle, we calibrate the parameter σ_j such that the model matches firms' earnings volatilities rather than their

Table 3
Calibration of corporate-level parameters.
Panel A reports the calibration values of parameters that do not vary among different credit ratings. τ_c denotes the corporate tax rate, τ_d the personal tax rate on dividends, and τ_i the tax rate on coupon income. Panel B shows some target parameters for individual firms' calibration by credit rating group. Debt-EBIT ratio is measured as (Short-term debt + Long-term debt) / EBITDA, corresponding to F/δ in the model. EBIT volatility is measured as the standard deviation of trailing five years of net revenue growth. Aggregate metrics reported in the table are the median values by credit rating. The underlying data are taken from Moody's Financial Metrics, a data and analytics platform that provides reported and adjusted financial data, ratios, models, and interactive rating methodologies.

Panel A: Parameters on tax rates and default losses

τ_c	τ_d	τ_i	b_0	b_1
0.35	0.12	0.296	0.113	11.846

Panel B: Calibration target by credit rating

Rating groups	Aaa	Aa	A	Baa	Ba	B
Debt/EBIT	0.90	1.43	1.85	2.61	3.25	5.03
EBIT volatility	9.82	8.60	10.52	12.16	16.58	13.64

[36] It is calibrated at 0.01 by BKS (2010), at 0.015 by Chen (2010), at 0.0176 by Benzoni et al. (2011), and at 0.02 by Wachter (2013).

[37] Jeong et al. (2015), who address the issue of the sensitivity of this estimate to the inclusion of ambiguity aversion, estimate the EIS based on the utility structure that we employ in our study. Their estimate is generally higher than one as well, but it is accompanied by large standard errors.

[38] This practice is advocated by Chen (2010), who argues for the use of nonmarket-based financial ratios in testing the implications of structural models.

[39] It indicates that the debt-EBITDA ratio does receive serious consideration, besides the conventional leverage ratio, in rating agencies' analytical process. Indeed, it is emphasized in these reports that "when Moody's does analyze financial ratios, it uses a multivariate approach."

Table 4
Model implications for default rates and credit spreads.

This table shows historical and model-implied ten-year default probabilities and ten-year credit spreads by credit rating group. Column (2) reports average cumulative issuer-weighted default rates estimated from Moody's Default & Recovery Database (DRD) from Moody's over the 1985–2015 sample period. Column (3) shows the model-implied stationary default probabilities; the corresponding ten-year survival probabilities are obtained by solving backward Kolmogorov equations on a grid of ambiguity levels. The model-implied 90% confidence intervals displayed in Column (4) are derived from 5000 simulations for the realized cumulative ten-year default rates. Columns (5) reports median corporate yield spreads as listed in Table IA.E7. The corresponding spread values corrected for the non-default component are shown in Columns (6) and (7); they are calculated based on the fractional sizes of the liquidity component estimated by Longstaff et al. (2005) and Chen et al. (2018), respectively. Column (8) reports model-generated credit spreads for a special case with $A_t \equiv \bar{A}$.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	10-yr default prob(%)			10-yr credit spread(bp)				
		Model		Total	Credit Component		Constant	Time-varying
	Data	Mean	90% intvl.	Observed	LMN	CCHM	ambiguity	ambiguity
Aaa	0.04	0.04	[0.00, 0.14]	77	39	33	8	32
Aa	0.45	0.42	[0.26, 0.75]	91	46	39	18	49
A	1.77	1.18	[0.87, 1.93]	117	66	67	53	78
Baa	3.66	4.24	[3.23, 6.39]	182	129	131	85	153
Ba	16.32	14.76	[13.41, 17.54]	298	247	241	164	225
B	37.64	40.13	[37.06, 45.96]	488	–	–	312	407

asset volatilities.[40] We find that the median debt-EBITDA ratios are strictly monotonic with ratings, while the earnings volatility shows a weak relation to ratings.

With values of F and σ_j determined for each credit rating, we solve for the total coupon payment C by following Leland (1994a)'s procedure. That is, the coupon is set so that the market price of newly issued debt D equals its principal value mF. As an illustration, consider Aaa-rated firms whose historical debt-to-EBITDA ratio is 0.90. When the value of corporate earnings starts at $\delta_0 = 100$, the face value of debt equals 90. Depending on the current level of ambiguity, the coupon payment $\bar{C}$ making the debt priced at par varies. For example, $\bar{C} = 5.61$ if A_0 is at its historical mean. To determine the unconditional expectation of model-generated credit spreads, we follow CCG (2009) by calculating the following conditional spreads for selected grids on A_0:

$$\bar{C}(A_0) = \left(\frac{1}{F} - \frac{1}{(\bar{C}(A_0) + mF)\mathscr{P}\mathscr{V}^{r+m}(A_0)}\right).$$

Thus, credit spreads are computed based on a nondefaultable bond with the same coupon rate and interest frequency. The reported model spread for each rating group is the population average over the steady-state distribution of A_t.

4.2.2. Default rates and credit spreads

Table 4 compares model-generated default probabilities and credit spreads to their empirical counterparts. In contrast to many existing studies, wherein attention is confined to investment-grade bonds, our goal is to match the targets for both investment-grade and speculative-grade debt. Both Huang and Huang (2012) and McQuade (2016) observe that, although the documented credit spread puzzle mainly involves the investment grades, when reconciling it, we should guard against creating "another puzzle in the other direction;" that is, the proposed model overpredicts credit spreads on high-yield bonds.

Column (2) in Table 4 reports the average cumulative default rates over the ten-year horizon for issuers of each (initial) credit rating. Based on Moody's Default & Recovery Database (DRD), which includes all defaults on rated bonds between 1970 and 2015, there are 22 (overlapping) ten-year intervals matching our sample periods. Column (3) shows that our model captures well the issuer-weighted average default rates across these 22 cohorts. Noteworthily, Feldhütter and Schaefer (2018) demonstrate that the standard approach in the literature—calibrating the model to the realized default rates—is subject to great sampling uncertainty, which may render the calibration results biased in favor of the credit spread puzzle. Our study, in contrast, departs from this approach by calibrating the model to debt ratios, as well as earnings volatilities, and thus endogenously generates default rates.[41] Nevertheless, we still perform a simulation to assess the impact of sampling uncertainty on the model-implied ex post default rates.

Based on the estimated state process, we first simulate aggregate variables over 31 years, which matches our data sample for default rates (1985–2015). At the start of each year (except for the last nine years), we form one cohort for each rating category and generate corporate earnings processes. For example, the Baa cohort initially consists of 765 identical firms, the average number of firms in Moody's Baa cohorts over the 1985–2015 period. At each point in time, we simulate 765 idiosyncratic shocks and combine them with aggregate output growth to obtain

[40] Since the financial ratios covered by Moody's Financial Metrics do not include earnings volatility, the volatility of net revenue is used as a proxy. An implicit assumption here is that the EBITDA margin is constant, and thus revenue and EBITDA have the same growth rate.

[41] Our firm-level calibration targets, the debt-EBITDA ratio and earnings volatility, correspond to the leverage ratio and asset volatility in value-based structural models. With the standard calibration approach, asset volatility is usually backed out from other calibration targets.

firm-specific growth rates. As such, we can collect the realized ten-year default frequency for each cohort and then calculate the average rating-specific default rates in this hypothetical economy.[42] Column (4) shows the 90% confidence bands for the default rates derived from 5000 simulations. These confidence intervals encompass the historical default rates for all rating groups, suggesting that the modeled stationary default rates in Column (3) are reconcilable with the historical ones given the statistical uncertainty associated with ex post default rates.

While matching the historical default rates indicates how well the model quantifies the credit risk under the physical measure, the model-generated credit spreads assess the model's performance under the risk-neutral measure. One important empirical consideration is what kind of measure for history spread levels should be used for comparison. The standard practice in the literature is to set the firm's leverage equal to the average leverage of firms in a particular rating class and then compare model-implied spreads with the (weighted) average actual spread over a period. However, David (2008) and BKS (2010) are critical of this "averages-to-averages" comparison, as it is subject to convexity bias in credit spreads.[43] In this paper, we adopt a "medians-to-medians" strategy: first, the model is calibrated to the median financial ratios for a given rating group; second, the model's prediction is compared with the median spread across all bonds in that rating category. Internet Appendix shows that this approach is largely immune to convexity bias.

The model's implications for credit spreads are reported in the right panel of Table 4. It is well-known that non-credit factors, such as illiquidity in the bond market, account for a sizable portion of yield spreads. Thus, in testing the model's predictions, we correct the target spreads for the fractional liquidity components estimated by Longstaff et al. (2005, LMN hereinafter) and by Chen et al. (2018, CCHM hereinafter). For example, the total yield for a typical Baa bond is about 182 basis points, as reported in Column (5) (also see Table IA.E7 in the Internet Appendix); the estimate by LMN (2005) of Baa bonds' liquidity fraction is 29%. The credit component should account for $182 \times 0.71 \approx 129$ basis points, as shown in Column (6). As expected, the liquidity fraction of the total spread becomes smaller as the rating quality of the bond decreases, while the absolute magnitude of the liquidity component decreases with the credit rating.

We find that the model successfully captures the spread level for both investment-grade bonds and high-yield bonds. Our result indicates that we do not force the model to match the historical spreads on high-quality bonds by overstating the amount of compensation demanded for per unit of default risk. In other words, this model improves the pricing of corporate debt by producing a higher level of uncertainty premium and proposing a better structure for the market price of uncertainty. To underline the effect of time variation in ambiguity, we also tabulate the credit spreads generated by time-invariant ambiguity. Column (7) in Table 4 indicates that the otherwise identical κ-ignorance model substantially underperforms our benchmark model, especially for investment-grade bonds. Indeed, the simplified model shows fairly limited improvement over the baseline model where the agent is ambiguity neutral. This finding is consistent with CCG (2009), who conclude that time-varying Sharpe ratios are essential for explaining the credit spread puzzle.

4.2.3. Theoretical insights and empirical verification

However, time-varying asset Sharpe ratios alone do not suffice to account for the high level of historical spreads—a point illustrated by the baseline model of CCG (2009), where the default boundary is constant. To explain the credit spread puzzle, Sharpe ratios need to show a strong correlation with the default time. In our model, it is accomplished by the optimal default boundary δ^* that increases with the level of ambiguity. This positive covariation in turn drives up the risk-neutral default probability far beyond its counterpart under the physical measure.

To demonstrate the effect of time-varying ambiguity on the firm's default decision, in Fig. 2 we plot δ^* as a function of A_t. We focus on a typical Baa-rated firm with the current earnings level normalized to 100. The figure shows that the default boundary is higher when the outlook for economic growth is more unclear than usual. In other words, management is more likely to exercise their default option earlier in the presence of a high degree of ambiguity. While Knightian uncertainty has little impact on the actual cash flows to the firm, it generates the comovement between the default probability and market prices of uncertainty.

Note that the ambiguity level also determines the asset valuation ratios. Numerically, its negative effect on the price-earnings ratio L^o dominates the positive effect on the earnings-based default boundary. Thus, the asset value at the time of default U^* is negatively related to A_t, as indicated by the dashed red line in Fig. 2.[44] The direction in which the default boundary moves actually constitutes an important empirical basis used in model validation and verification. For instance, Davydenko (2012) finds that the value-based boundary is procyclical; this result supports the prediction of Chen (2010) and BKS (2010). Following

[42] Note that the default boundary is the same for these 765 firms because they start with the same initial debt-earnings ratio and are exposed to the same degree of ambiguity throughout the simulated sample. Feldhütter and Schaefer (2018) find that the downward bias in realized default rates is mainly attributable to the presence of systematic risk. Systematic risk is present in our simulated economy as well: first, the earnings process for individual firms contains a systematic component, as specified in Eq. (15); second, the variation in the (fractional) default boundary is driven by ambiguity shocks, which is common to all firms in the economy. Therefore, untabulated results indicate that our simulated ex post default rates share the same pattern as those of Feldhütter and Schaefer (2018); that is, their distribution is skewed to the right and their median is below the mean.

[43] An early version of Feldhütter and Schaefer (2018) raises the same point and attempted to quantify the convexity bias. Huang and Huang (2012) and CCG (2009) find that if structure models are calibrated to the historical default loss experience, the convexity effect is small and makes the historical spreads even more of a puzzle.

[44] This countermovement of default boundaries based on cash flows and the asset value is also documented by Hackbarth et al. (2006), Chen (2010), and BKS (2010).

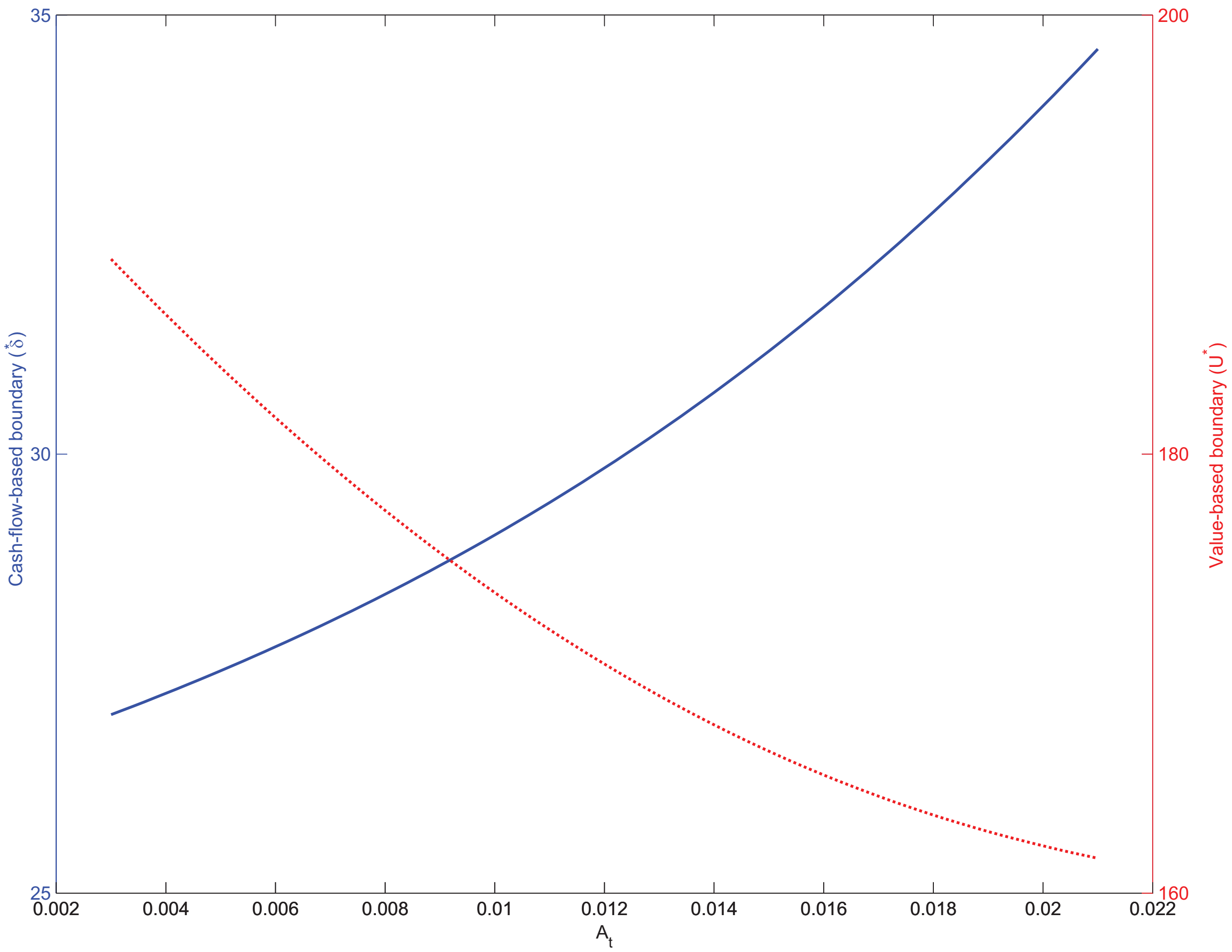

Fig. 2. Optimal default boundary as a function of the ambiguity level.
This figure plots the optimal default barriers for a typical Baa-rated issuer against the level of ambiguity. The solid blue line shows the default boundary in terms of the corporate earnings $\delta^*(A)$. The dashed red line shows the unlevered asset value at default $U^*(A)$. The optimal boundary is computed assuming the firm's cash flow starts at $\delta_0 = 100$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

his methodology, we test our model's implication by regressing sample estimates of boundary levels (based on firms that defaulted on their public bonds) on the ambiguity measure along with other theoretical determinants.[45]

The regression results are reported in Table 5, where the default boundary is expressed as a fraction of the face value of outstanding debt. To correct for selection bias, we use Heckman (1976, 1979)'s procedure of two-stage regressions.[46] The univariate regression indicates that a high degree of ambiguity lowers the level of the value-based boundary, which is consistent with the model's implication. The ambiguity factor alone explains 4.9% of variations in observed boundaries, which is substantial compared to the R^2 values reported by Davydenko (2012). This effect is significant even after controlling for other determinants suggested by structural models. Moreover, the ambiguity measure seems to crowd out the significance of the GDP growth rate, which is used to proxy for macroeconomic conditions. This result implies that the ambiguity level contains relevant information on corporate default behaviors, above and beyond that contained in the business cycle.

4.3. The levered equity premium and other asset pricing implications

CCG (2009) describe calibration exercises on modeled credit spreads as an out-of-sample test of potential solutions to the equity premium puzzle. Given this insight, this section brings the focus back to the equity premium by

[45] Asset values at default are estimated based on observed market prices of debt and equity as well as empirical estimates of the risk-neutral default probability. In this study, default boundaries are obtained as a by-product of the estimation of the default cost. Internet Appendix D reports summary statistics of the estimated default boundary. We find that, on average, the location of the default boundary is measured at 68% of the book value of debt, a value very close to the 66% reported by Reisz and Perlich (2007) and Davydenko (2012).

[46] In Table 5, the inverse Mills ratio term turns out to be statistically insignificant in all nine regression models listed. Therefore, the null of no selection bias cannot be rejected, and t-statistics are computed based on unadjusted standard errors.

Table 5
The effect of ambiguity on the default boundary.
This table reports cross-sectional regressions of the empirically observed boundary on the ambiguity level and other theoretical determinants. The default boundary is defined as the value of a firm's assets at default, scaled by the face value of debt. The regression coefficients are estimated with the Heckit model (Heckman, 1976; 1979). $1/Mills$ denotes the inverse Mills ratio obtained from the first-stage regression, which includes all independent variables at the second stage and the quick ratio. The values of t-statistics are reported in parentheses. The sample period spans from 1983 to 2010.

$\widetilde{A}_t$	−0.198	−0.146	−0.193	−0.175	−0.150	−0.192	−0.203	−0.194	−0.184
	(−3.450)	(−2.053)	(−2.796)	(−3.648)	(−2.606)	(−3.305)	(−3.551)	(−3.446)	(−2.998)
$Maturity_{j,t}$		−0.091							
		(−6.723)							
rf_t			−0.452						
			(−0.211)						
$Vol_{j,t}$				−0.286					
				(−2.781)					
$Coupon_{j,t}$					−2.845				
					(−3.232)				
$Cost_{j,t}$						0.322			
						(3.040)			
$Payout_{j,t}$							0.395		
							(0.553)		
GDP_t								0.038	
								(0.837)	
$Tax_{j,t}$									0.245
									(1.088)
$1/Mills$	−0.082	−0.105	−0.075	−0.047	−0.061	−0.119	−0.084	−0.086	−0.132
	(−0.793)	(−1.172)	(−0.682)	(−0.679)	(−0.727)	(−1.073)	(−0.834)	(−0.845)	(−1.562)
Const.	0.966	0.965	0.979	1.099	1.095	0.713	0.927	0.936	0.706
	(4.622)	(4.164)	(4.769)	(4.230)	(4.558)	(2.635)	(4.582)	(4.393)	(4.589)
Adj R^2	0.049	0.208	0.045	0.113	0.086	0.165	0.124	0.048	0.038
N	230	230	230	230	225	114	227	230	144

Table 6
Simulated and sample moments of equity return and risk-free rate.
This table shows historical and model-implied unconditional moments of the equity return and the risk-free rate. In the "Data" section, Column (2) reports the return on the NYSE/Amex/Nasdaq value-weighted index and Column (3) contains the median descriptive statistics of monthly returns on all Baa-rated stocks listed on CRSP. The "Time-varying ambiguity" section reports the implications of our benchmark model, while the "Constant ambiguity" section shows a special case with $A_t \equiv \bar{A}$. Columns labeled "unlevered" and "levered" present the results on unlevered and levered perpetual claims to aggregate output. Column (8) corresponds to a typical Baa firm whose earnings growth rate is also subject to idiosyncratic shocks. The sample period spans from 1985 to 2010.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Data		Constant ambiguity		Time-varying ambiguity		
	Portfolio	Individual	Unlevered	Levered	Unlevered	Levered	Individual
$E(r_{e,t}-r_t)$	6.13	8.98	1.11	2.09	4.13	5.94	7.81
$\sigma(r_{e,t})$	16.58	32.97	1.18	2.67	6.72	12.39	26.63
$E(r_{e,t}-r_t)/\sigma(r_{e,t}-r_t)$	41.49	28.33	93.97	78.62	73.75	48.60	29.24
$E(r_t)$	1.21		1.70			1.18	
$\sigma(r_t)$	1.18		0			0.92	

examining our model's ability to explain relevant empirical regularities. To this end, we focus on levered equity as a residual claim to earnings. As emphasized by BKS (2010), this approach introduces a default-risk dimension into the equity premium and preserves a direct link between asset pricing and firm-level decisions.

Column (2) of Table 6 reports real equity returns constructed using the CRSP value-weighted index. While the statistics are computed from the data for the 1985–2010 period, they appear fully consistent with the numbers for the entire postwar sample. Accordingly, model-implied moments are computed based on the unlevered and levered claims to aggregate output O_t.[47] For levered equity, the initial ratio of debt to cash flow is calibrated at 2.61, the median value for all Baa firms. This is motivated by the finding of Avramov et al. (2007a) that the average rating for 3,578 public firms rated by S&P is approximately BBB.

Column (7) shows that the model-implied premium and volatility of levered equity are 5.9% and 12.4%, respectively, matching the data reasonably well.[48] Both are sub-

[47] To obtain the model's implications for the levered equity, we run 1000 simulations based on the Euler discretization. Each simulation generates 26 years' worth of daily observations. Simulating the model at lower frequencies makes the chance of getting negative values for A_t higher than 0.001.

[48] While equity volatility is a bit below that found in the data, it falls within two standard errors of the empirically estimated volatility (the standard error of annualized volatility is estimated at 2.78%, based on the Newey–West adjustment with the lag length equal to six quarters). Also, because A_t follows a CIR process, its conditional standard deviation rises

stantially greater than their counterparts for unlevered equity in Column (6) since the nature of levered equity as a residual claim increases the uncertainty in cash flows to shareholders. For comparison, we also present the implications of a κ-ignorance model. Columns (4) and (5) show that the restriction $A_t \equiv \bar{A}$ significantly lowers both the unlevered and levered equity premiums. To understand the contribution of time variations in A_t, consider the expression for the former (which is in a closed form):

$$\frac{1}{dt}E_t\left[\frac{dU((1-\tau_e)\delta_t, A_t)}{U((1-\tau_e)\delta_t, A_t)}\right] - r_t + \frac{(1-\tau_e)\delta_t}{U((1-\tau_e)\delta_t, A_t)}$$
$$= \gamma\sigma_c\sigma_o\sigma_{oc} + \frac{\sigma_o\sigma_{oc}}{\sigma_c}A_t + (1-\theta)\eta_1\xi_1\sigma_a^2 A_t. \quad (24)$$

The first term in Eq. (24) reflects the standard risk exposure in a CRRA setting, and the second term represents the pure ambiguity premium, capturing the fact that Knightian uncertainty about the future payoffs is a first-order concern. These two terms are retained even in the κ-ignorance case. Numerically, however, they are subordinate to the third term, which is caused by the uncertainty associated with temporal variation in the ambiguity level, or more fundamentally, by the interaction of learning and ambiguity aversion. The learning model in Internet Appendix A sheds light on this result: the ambiguity level moves slowly with signals as the agent responds asymmetrically to good and bad news. The resultant permanence of the ambiguity process, as captured by the model parameter κ, raises the absolute values of η_1 and ξ_1 and thus amplifies the third term. Quantitatively, this is reflected in the 1.1% unlevered equity premium with time-invariant ambiguity, against the 4.1% with time-varying ambiguity. Also, the κ-ignorance model cannot generate excess equity volatility, as it posits that the volatility of unlevered equity equals that of dividends.

Our discussion so far leaves one important question unanswered: to what extent is the market index representative of a hypothetical Baa-level firm that is only exposed to systematic shocks? In response to this concern, we calibrate the model using the method described in Section 4.2.1: the firm's earnings follow Eq. (15) with the total volatility calibrated to 12.16%, which is the median volatility for Baa firms. These calibration results are then compared with the median values among all Baa firms.[49] As indicated by the results in Column (3), a typical Baa firm has both a higher mean excess return and a higher return volatility than the market index. This finding is in qualitative agreement with our model's implication shown in Column (8), as the inclusion of idiosyncratic cash flow risk increases the default risk embedded in levered equity.[50] Quantitatively, the model-implied equity premium and volatility for an individual Baa firm are largely consistent with their data counterparts. Moreover, the model reproduces the stylized fact that individual firm volatility is approximately twice the level of market volatility (CCG, 2009).

As demonstrated in CCG (2009), a key statistic to be matched in explaining the credit spread puzzle is the (rating-specific) average Sharpe ratio. In a departure from their procedures, where the firm-level Sharpe ratio is an explicit calibration target, we calibrate our model to corporate earnings volatility, which endogenously produces Sharpe ratios for individual firms. Based on equity returns, the median Sharpe ratio of Baa-rated firms is about 0.28, which is remarkably consistent with the 0.29 predicted by the model. If we consider only the systematic component in a Baa firm's cash flow, the model's prediction is roughly comparable to the Sharpe ratio for the market portfolio. However, when the ambiguity level is further restricted to a constant, the model-implied Sharpe ratio jumps to about 0.79. This reveals that the restricted model delivers unrealistically high returns (on levered equity) per unit of standard deviation.

The last two rows of Table 6 present the model's predictions about the risk-free rate. We find that the model-generated mean (1.18%) is almost identical to the historical counterpart (1.21%), but a degenerate set of priors implies a significantly higher level. As shown in Appendix B, the restriction $A_t \equiv \bar{A}$ lessens the impact of Knightian uncertainty, which therefore works only through misspecification doubts about expected consumption growth (or equivalently, only through the intertemporal substitution channel). By the same token, the κ-ignorance model implies no variation in the risk-free rate, whereas ours generates an annualized volatility of 0.92%. Hence, to match the empirical moments of the risk-free rate, there needs to be uncertainty about future levels of ambiguity, which contributes to the investors' precautionary savings motive.

4.4. Historical variations of asset prices

While several extant preference-based models capture levels of the equity premium and credit spreads (CCG, 2009; Chen, 2010; BKS, 2010), their ability to match the historical behavior of asset prices has not been fully demonstrated. An important advantage of our model lies in its ability to identify the time series of financial variables based on the empirical measure for ambiguity and the calibrated model. A comparison of these time series with their historical counterparts provides an intuitive way to evaluate the model's performance.[51] Fig. 3 presents the

with the square root of its level. Consequently, equity volatility is a nondecreasing and concave function of the ambiguity level. Given that the equity price is nonincreasing in A_t, the model can reproduce "the leverage effect" (Black, 1976).

[49] The median is obtained from a July 1985 to December 2010 sample of BBB-rated stocks listed in the CRSP. The sample's beginning is determined by the first date for which credit ratings by S&P are available on the Compustat database. To be included in the sample, the stock must have at least 12 consecutive monthly return observations.

[50] Note that in the presence of idiosyncratic risk, the ambiguity level plays a limited role in determining the condition volatility of equity returns—a pattern that is consistent with empirical evidence that volatility asymmetry is generally stronger for aggregate market index returns than for individual stocks (Kim and Kon, 1994; Tauchen et al., 1996; Andersen et al., 2001).

[51] Appendix D considers other aspects of asset dynamics — the long-horizon predictability of equity returns and yield spreads as well as their correlations with consumption growth — and the performance of our models in capturing relevant empirical regularities.

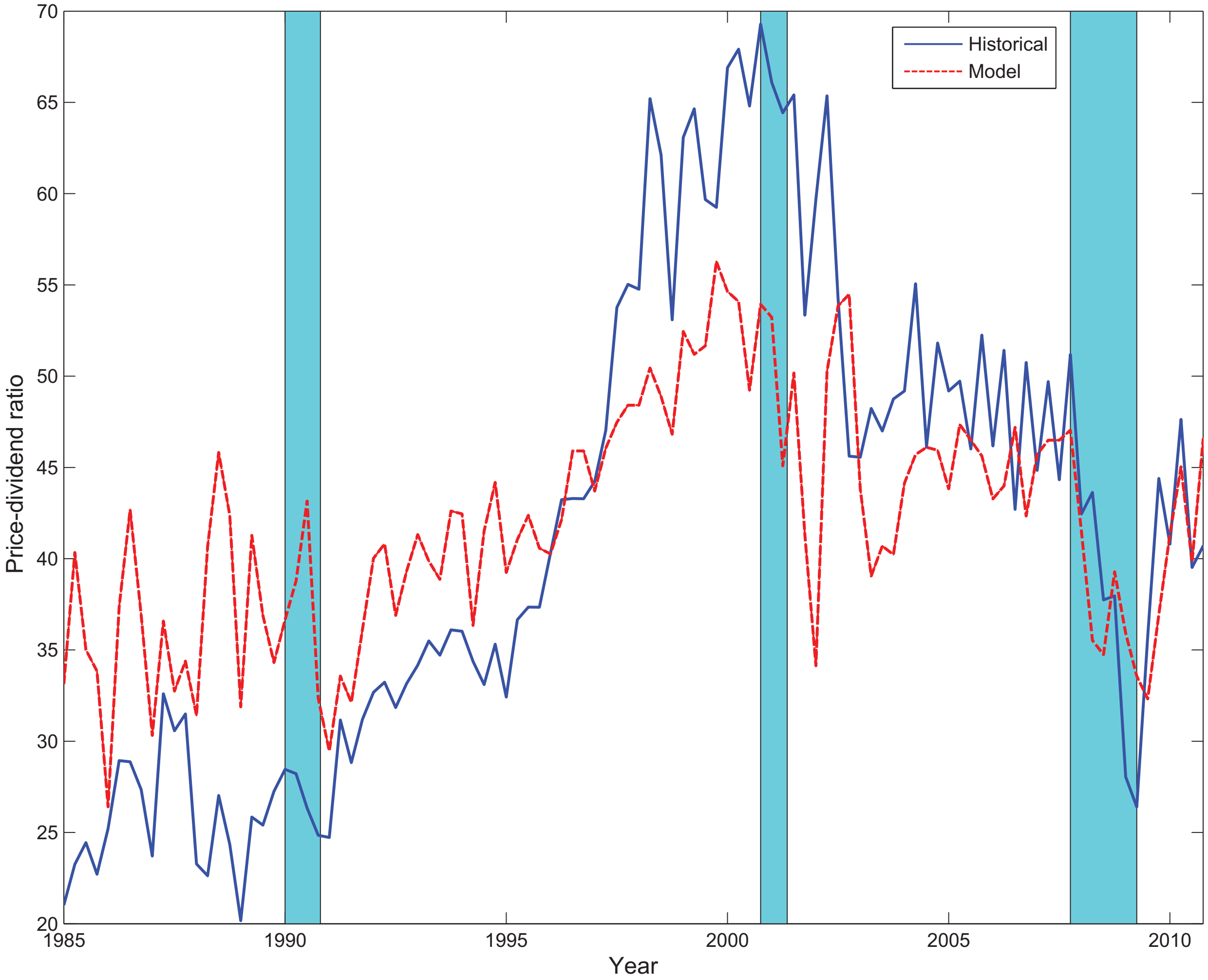

Fig. 3. Historical and modeled price-dividend ratios.
This figure displays the realized time variations in the price-dividend ratio and the model's prediction. The solid blue line shows the historical price-dividend ratio on the NYSE/AMEX/NASDAQ value-weighted index. The dashed red line shows the model-generated P/D ratio computed based on historical values of the ambiguity measure $\widetilde{A}_t$. Shaded bars denote months designated as recessions by the National Bureau of Economic Research. The sample period spans from 1985 to 2010. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

model's prediction for the price-dividend ratio of levered equity and the actual price-dividend ratio on the CRSP value-weighted index. It is unclear how well the index represents a levered claim to aggregate output; thus attention must be paid to the historical fluctuations in (rather than the level of) the model-implied ratios. We conclude that the model provides an accurate account of serial variation in the P/D ratio: it can reproduce the countercyclical behavior of stock prices, as documented in Campbell and Shiller (1988) and Fama and French (1989), and it captures shorter-term fluctuations within each business cycle. This high-frequency covariation incarnates an important model implication: changes in the ambiguity level can lead to significant responses of asset prices, even without shocks to the economic fundamentals. Indeed, the correlation between model-predicted and historical P/D ratios is 75%, which is much higher than the 21% reported by CCG (2009), who study a different and shorter sample period.

As Fig. 4 suggests, the model is particularly useful for fitting the dynamics of historical credit spreads. The ambiguity-implied Baa spread is depicted as the dotted red line. The blue and green lines correspond to Barclays aggregate Baa bond spread and Merrill Lynch BBB seven-to ten-year option-adjusted spread, respectively. Again, the model's prediction captures the countercyclical pattern of historical spreads, and it exhibits similar dynamics within each business cycle. Indeed, its correlations with both yield spread indices are higher than 82%. Equivalently, it can explain more than 67% of the variations in observed spreads, and the regression residuals show little evidence of autocorrelation. As a benchmark, the corresponding correlation coefficients generated by CCG (2009)'s and Buraschi et al. (2014)'s models are 72% and 41%, respectively.[52] In Fig. 4,

[52] The Baa-Aaa yield spread predicted by David (2008)'s model achieves a 80% correlation with its historical counterpart. However, this result is

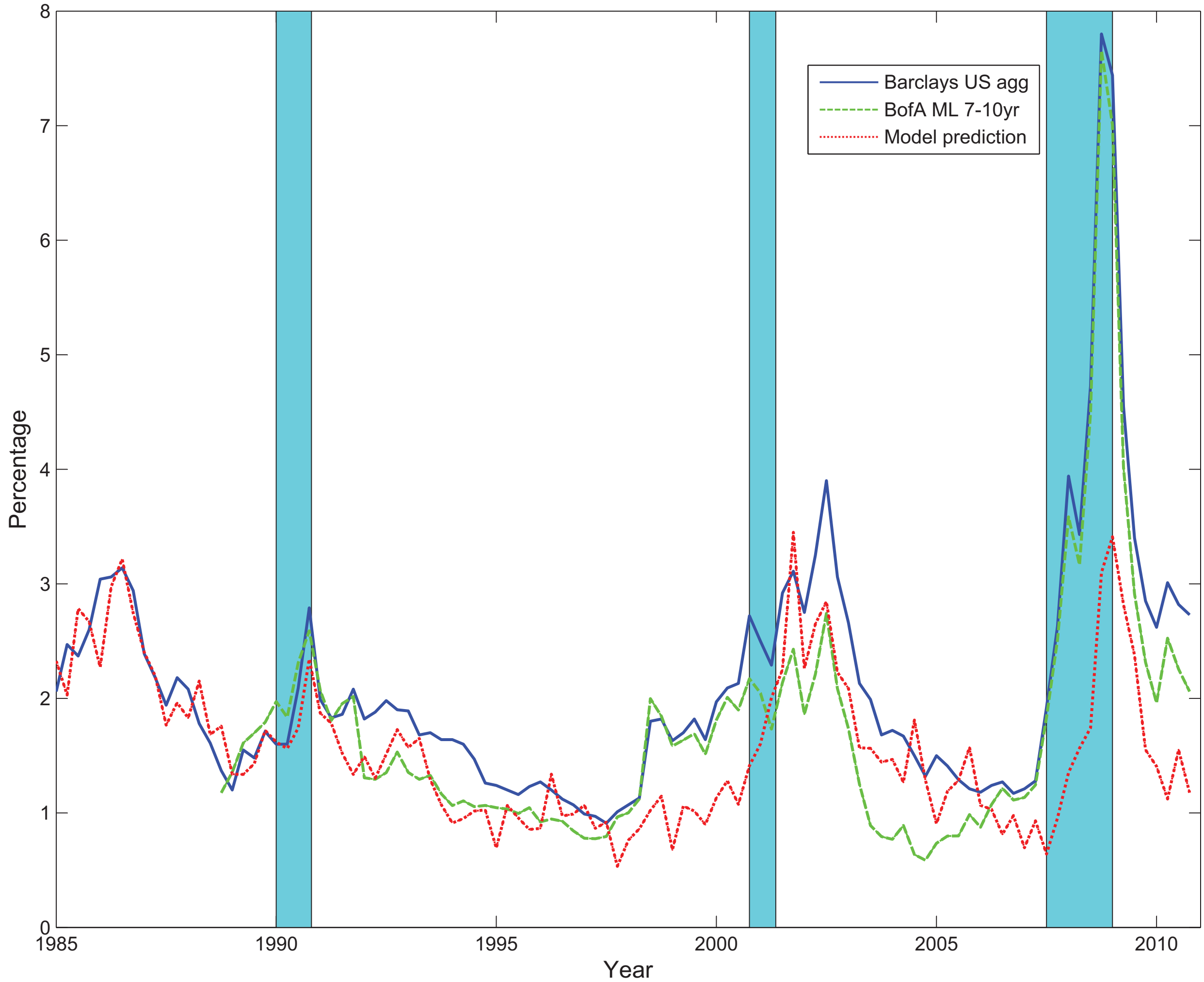

Fig. 4. Historical and modeled spreads in yields between baa and default-free bonds.
This figure displays the realized time variations in ten-year credit spreads for Baa bonds. The solid blue line shows the historical credit spread of Barclays US aggregate corporate Baa bond index, and the dashed green line corresponds to Bank of America Merrill Lynch US corporate BBB option-adjusted spread. The dotted red line shows the model-generated credit spread computed based on historical values of the ambiguity measure $\widetilde{A}_t$. Shaded bars denote months designated as recessions by the National Bureau of Economic Research. The sample period spans from 1985 to 2010. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the largest discrepancies between the observed and model spreads occur around financial crises. This result is to be expected given that the liquidity shortage tends to play an important role in these episodes, which are not modeled in this paper. In summary, whereas the evidence presented in Fig. 1 is merely suggestive of the role of ambiguity in reconciling the credit spread controversy, that provided in Fig. 4 is considerably more conclusive.

5. Conclusion

This paper demonstrates the effects of time variation in ambiguity on equilibrium asset pricing in the context of corporate yield spreads and the levered equity premium. Qualitatively, it exerts both first-order and second-order effects on asset premiums. The first-order effect is directly bound to the difference between the true consumption growth and the current worst-case belief used by the agent to evaluate assets; the second-order effect reflects the agent's desire to hedge against permanent shocks to the degree of ambiguity. Quantitatively, both channels are essential for the model to match key statistics of historical equity returns and credit spreads. These theoretical implications are supported by the empirical evidence for the predictability of realized asset premiums produced by our novel measure of the ambiguity level.

Our ambiguity-based model provides a better fit with historical variations in stock prices and credit spreads than do other existing frameworks that price equity and corporate debt jointly. Our results suggest that economists

based on a unique calibration method: the model is fed with the time series of a market-based leverage measure — the market value of assets divided by the book value of debt.

should look beyond the business cycle when characterizing the dynamics of asset prices. In the framework of Chen (2010) and BKS (2010), the model's fit to the historical yield spreads is sacrificed through a discretization of expected consumption growth and consumption volatility. In this manner, they are able to deliver closed-form solutions to equity and debt prices with dynamic capital structure. Given this insight, an extension of our model to allow for endogenous capital structure presents an interesting topic for future research. Agency costs and transaction (debt retirement and reissuance) costs, as considered by Leland (1998) and Strebulaev (2007), can be incorporated into the extended model as well.

Another interesting direction for future research is to consider the implications of our model for derivative pricing. For instance, one can introduce secondary market search frictions (He and Milbradt, 2014) and evaluate the model's performance in fitting bond-CDS spreads (the non-default components of corporate yield spreads). Also, our model can be extended to shed light on stylized facts about option-implied volatility, that is, the "volatility smirk" and the variance premium, as studied by Drechsler (2013).[53]

Appendix A. More properties of the proposed ambiguity measure

This appendix supplements Section 3.1 with more details on the properties of $\widetilde{A}_t$. The first two sections draw a comparison of $\widetilde{A}_t$ with other empirical measures based on the dispersion of survey forecasts. Specifically, Section A.1 covers alternative ambiguity measures, and Section A.2 considers empirical proxies proposed for heterogeneous beliefs models. The relations of all these measures (including $\widetilde{A}_t$) with macroeconomic volatility are examined in Section A.3.

A.1. Alternative measures of ambiguity

In this section, we place the proposed empirical measure in the context of relevant literature by comparing it with alternative ambiguity measures used in existing studies. AGJ is arguably the first to test the link between time-varying ambiguity and asset pricing by constructing an index of Knightian uncertainty. The key "ambiguous" variable in their model is the expected real return on the market portfolio in excess of the risk-free rate. To construct a relevant ambiguity measure, they collect from the SPF panelists' predictions on several economic variables, including corporate profits after taxes, GDP deflator, and nominal T-bill rates. With these variables, they impute each participant's forecast of real market return based on Gordon's formula and then proxy the level of ambiguity by a weighted variance of forecasts. By the same spirit, Drechsler (2013) proposes a measure based on the cross-sectional standard deviation in SPF forecasts of real GDP growth, which is used to provide empirical support for a link between ambiguity and the variance premium.

It is noteworthy that both measures are conceptually consistent with the way they construct the set of alternative beliefs. Specifically, they set a upper bound for the (growth rate of) relative entropy,[54] which, in this paper's setting, is equivalent to a upper bound for the norm of the density generator (ϑ_t). This approach to restricting the amount of ambiguity is adopted in Hansen and Sargent (2008)'s robust control framework. With multiple-priors preferences, in contrast, the belief set is restricted by a rectangular set of density generators. This rectangularity inspires our use of cross-sectional range, rather than variance (standard deviation), as the ambiguity measure.

The empirical measure used in Ilut and Schneider (2014) is another example of using the interdecile range of survey forecasts to infer the dynamics of ambiguity in multiple-priors models. In an early version (Ilut and Schneider, 2011), their measure is based on the difference between the 75th percentile and the 25th percentile in each cross-section of forecasts. The resulting interquartile range is essentially the Federal Reserve Bank of Philadelphia's definition of "forecast dispersion" and is included in its SPF database. In Table A.1, we make a comparison of our ambiguity measure with the three alternative ones as aforementioned.

Panel A provides a description for each empirical measure and particularly highlights their forecasting performance. Because of the space limitations, only the p-values obtained from univariate regressions of equity returns are reported. We focus on stocks here, as almost all asset pricing models carry implications for the equity premium but not necessarily for other asset classes. We find that the AGJ measure is the only one (besides our measure $\widetilde{A}_t$) with significant forecasting power. The correlation between their measure and ours is not remarkable, at approximately 25%, which is not surprising given that they are meant to capture the agent's confidence about different economic quantities. However, they do share some notable similarities in the way they are constructed. In particular, both their weighted variance and our interdecile range allow for the possibility of minimizing the impact of extreme forecasts.[55] As discussed in Epstein and Schneider (2003), the difference between the multiple-priors and robust control models is a matter of alternative restrictions on the belief sets and on updating rules and which type of restriction to be imposed in ambiguity modeling "will typically depend on the application." Our result suggests that, at least in

[53] Drechsler (2013) specifies an exogenous dividend process and treats equity as an unlevered claim to the dividend. Therefore, his model could be calibrated less aggressively than he does to capture qualitative features of the equity premium and variance premium. Also, if we embed a structural credit model into his framework with the firm-level parameters the same as in our model, we find that his calibration tends to overpredict corporate default rates and credit spreads when the default barrier is set endogenously. When the default boundary is set as constant (at 68% of the book value of debt), the model-generated yield spreads become insufficient to match the empirical credit spreads.

[54] Relative entropy is the Kullback–Leibler distance between an alternative probability measure and the reference measure.

[55] When computing the weighted variance of forecasts, AGJ assign the weight to each forecast based on a discretized beta distribution. They force both shape parameters to be equal and greater than one such that the weight is symmetric and heavily penalizes observations lying on the left and right tails.

Table A1
A comparison of measures based on the dispersion of survey forecasts.
This table compares features of empirical measures that are constructed from the distributions of survey forecasts to proxy for ambiguity perceived by the representative agent or disagreement between heterogeneous agents. These measures include the ones examined in Anderson et al. (2009), Drechsler (2013), an early version of Ilut and Schneider (2014), Ilut and Schneider (2011), Buraschi and Jiltsov (2006), Buraschi et al. (2014), Yu (2011), and this article. The comparison table is divided into three panels. Panel A summarizes the formation of these measures and corresponding economic concepts in the model. The last row of this panel presents the *p*-value each measure achieves when it is used to forecast one-quarter-ahead excess return on the CRSP value-weighted stock index. Panel B provides pairwise Pearson correlation coefficients for these empirical measures. Panel C focuses on the correlation between these measures and proxies for time-varying volatility.

	This article	Anderson et al. (2009)	Drechsler (2013)	Ilut and Schneider (2011)	Buraschi and Jiltsov (2006)	Buraschi et al. (2014)	Yu (2011)
Panel A: Summary of measures and their corresponding models							
Type of model	ambiguity aversion	ambiguity aversion	ambiguity aversion	ambiguity aversion	heterogeneous beliefs	heterogeneous beliefs	
The economic quantity about which the agent(s) feel uncertain / disagree	expected consumption growth	expected return on the market index	(a) drift of expected consumption growth; (b) expected consumption volatility; (c) intensity of jumps in expected consumption growth and in consumption volatility	expected innovation in TFP	expected consumption growth	expected consumption growth	
Statistics underlying the measure	interdecile range	weighted variance	standard deviation	interquartile range	standard deviation	mean absolute deviation	standard deviation
Data source	Blue Chip	SPF	SPF	SPF	SPF	I/B/E/S	I/B/E/S
p-value from univariate predictive regressions	0.003	0.082	0.183	0.143	0.280	0.037	0.096
Panel B: Cross-sectional correlation among measures							
Anderson et al. (2009)	0.250						
Drechsler (2013)	0.479	0.192					
Ilut and Schneider (2011)	0.634	0.234	0.665				
Buraschi and Jiltsov (2006)	0.625	0.144	0.521	0.671			
Buraschi et al. (2014)	0.068	0.166	−0.004	0.032	0.216		
Yu (2011)	−0.038	0.024	−0.063	−0.046	−0.004	0.261	
Panel C: Cross-sectional correlation with volatility proxies							
$EV_{GDP,t}$	−0.105	0.005	−0.057	−0.240	−0.274	−0.163	−0.147
$RV_{IP,t}$	−0.170	−0.217	−0.068	0.018	−0.198	−0.240	−0.085
$RV_{M,t}$	0.302	0.053	0.296	0.354	0.459	0.205	−0.068
VIX_t	0.391	0.021	0.350	0.401	0.413	0.050	−0.099
p-value from predictive regressions when VIX_t is controlled for	0.004	0.119	0.245	0.243	0.156	0.048	0.059

testing the implications of ambiguity for asset premiums, the choice about the modeling framework is less important than the robustness of the adopted empirical measure to outlying observations.

A.2. Measures of disagreement among heterogeneous agents

Outside of the literature of Knightian uncertainty, the dispersion of survey forecasts is also used to proxy for the level of disagreement in heterogeneous belief models. Interestingly, this stream of research contains at least two different approaches that imply opposite signs about the effect of belief heterogeneity on the equity premium. Specifically, the models of Miller (1977) and Chen et al. (2002) focus on the interaction between disagreements and short sales constraints and suggest that greater disagreement leads to lower subsequent returns. Empirically, Diether et al. (2002) find supportive evidence for this proposition by showing that earnings forecast dispersion negatively predicts future stock returns. The second approach, on the other hand, considers agents with identical preferences but different beliefs about economic prospects. In the absence of trading frictions, the equilibrium discount rate increases with the level of disagreement. This implied positive effect of forecast dispersion on the equity premium is supported by the time-series evidence in Anderson et al. (2005) and by the cross-sectional evidence in Buraschi et al. (2014, BTV hereinafter).

In this section, we attempt to distinguish our ambiguity variable $\widetilde{A}_t$ from measures for differences in beliefs. Since we do not explore in our model the implications of cross-sectional variation in the uncertainty about individual security, we focus our comparison on aggregate disagreement indices.[56] Regarding the disagreement-with-short-sales-constraint approach, we consider Yu (2011)'s measure, which can be regarded as an aggregation of Diether et al. (2002)'s or AGJ (2005)'s individual-level measures and a proxy for the disagreement about aggregate corporate earnings. Among studies with the second approach, we select the measures constructed by Buraschi and Jiltsov (2006) and BTV (2014). The former measure is the first principal component of differences in beliefs about six economic variables, where the difference in beliefs is defined as the cross-sectional standard deviation of SPF forecasts. The latter is similar to Yu (2011)'s measure in the sense that it is constructed bottom-up by aggregating disagreements regarding the earnings of individual firms. The biggest difference between them arises from the underlying statistics: Yu (2011) uses the standard deviation of analyst forecasts, whereas BTV (2014) adopt the mean absolute difference (the average of absolute differences between each pair of earnings forecasts).

Panel A of Table A.1 indicates that the measure of BTV (2014) is highly significant in forecasting one-quarter-ahead market returns, and Yu (2011)'s measure exhibits statistical significance at the 10% level. Unreported results show that both slope coefficients are negative, which is not surprising given that both measures are constructed bottom-up using the same dataset (I/B/E/S). While this is consistent with Yu (2011)'s interpretation, it is opposite to the implications from BTV (2014)'s model.[57] In contrast, the sign of the regression coefficient is positive for the Buraschi–Jiltsov measure, even though the null of no predictability cannot be rejected at any conventional significance level.

While both our ambiguity measure and aforementioned disagreement measures are extracted from forecast dispersion, they have distinct properties, the most noticeable being the underlying statistics and predictive ability. We find that the range-based measures seem to be exclusively used in studies with multiple-priors utility, for the reason explained in the last subsection. One may argue that standard-deviation-based measures for disagreements—which are particularly relevant when we assume that each agent corresponds exactly to one participating forecaster (AGJ, 2005)—could also be used to proxy for Knightian uncertainty in multiplier models (AGJ, 2009; Drechsler, 2013) and thus are conceptually related to our measure $\widetilde{A}_t$. However, those measures show either a low (or even negative) empirical correlation with $\widetilde{A}_t$ or limited explanatory power for asset premiums.

Both findings merit further discussion. First, the negative correlation coefficient between $\widetilde{A}_t$ and Yu (2011)'s measure, as displayed in Panel B, reinforces his supportive evidence for the Miller (1977) conjecture (about the negative effect of disagreement on expected stock returns). Second, at first glance the insignificance of the Buraschi–Jiltsov measure in predicting equity returns is somewhat inconsistent with their model, which implies that differences in beliefs should be a significant determinant of the equity premium. If we interpret this result from a difference perspective, it nevertheless may actually reflect a key difference between two classes of models. In the models of Buraschi and Jiltsov (2006) and BTV (2014), it is the interaction between the degree of belief heterogeneity and the consumption share of the optimist, instead of either component alone, that drives the variation in equity premium. This interaction is absent in models with a representative ambiguity-averse agent. Therefore, the missing-variable issue could potentially explain why their measures are insignificant or have the wrong sign in univariate predictive regressions.

A.3. Relation with time-varying volatility

Another possible interpretation of forecast dispersion that deserves discussion is that it captures macroeconomic volatility. Theoretically, Bansal and Yaron (2004) and Bansal et al. (2014, BKSY hereinafter) posit that an increase in volatility is associated with a rise in discount rates, a prediction similar to our model's implication for ambiguity. Hence, it is important to draw a distinction between measures of time-varying ambiguity and those of time-varying volatility. In this section, we perform a comparative analysis of these empirical measures by considering both macroeconomic and return-based proxies for volatility as used in BKSY: the realized variances of industrial production growth rates and market returns.

For robustness, we examine as well the difference of $\widetilde{A}_t$ from Bansal and Shaliastovich (2013)'s measure of the expected variance of consumption growth. That is, they estimate innovations in the SPF consensus (mean) forecast of real GDP growth and then regress annual sum squares of future innovations on the current explanatory variables to obtain the time series of expected (fitted) consumption variance. Finally, given that both the BKSY ex post measure and the Bansal–Shaliastovich ex ante measure are about volatility under the physical measure, we also include in our comparison the option-implied variance (VIX) as a proxy for the Q-measure volatility.

Panel C of Table A.1 shows that our ambiguity measure does not have a particularly high correlation with any of the four volatility proxies. The correlation coefficients with respect to macroeconomic-based volatility proxies are even negative.[58] More importantly, $\widetilde{A}_t$ preserves the same level of statistical significance in predicting equity returns when we control for VIX, the volatility proxy with the greatest predictive power.[59] It implies that $\widetilde{A}_t$ contains information

[56] In other words, we do not consider disagreements about the earnings of individual stocks (Diether et al., 2002) nor disagreement *factors* constructed as the returns from longing high-dispersion stocks and shorting low-dispersion stocks (AGJ, 2005).

[57] BTV (2014) find that their measure represents a positively priced risk in the cross section of stock returns, but they do not test the return predictability with this measure.

[58] Results regarding return-based volatility proxies should be down-weighted, in the sense that based on the model presented in Section 2, the volatility of levered equity is a nonlinear function of the ambiguity level. Therefore, even if consumption growth exhibits time-varying volatility, it is intertwined with time-varying ambiguity in the determination of equity volatility in equilibrium.

[59] The R^2 from univariate regression of excess stock returns on VIX is about 0.023, and the t-statistic for VIX is 0.927.

regarding risk premiums independent of stochastic volatility.

The results for other measures (based on forecast dispersion) as discussed above are also reported for completeness. We find that among these six measures, the one constructed by Buraschi and Jiltsov (2006) displays the highest correlation with time-varying volatility. For those with significant forecasting power for equity returns, their significance is generally unaffected by the inclusion of VIX. This finding suggests that volatility drives time variations in the asset premium through a risk channel that is distinct from those for time-varying ambiguity and heterogeneous beliefs.

Appendix B. Equilibrium prices and affine approximation

Given the value function of the representative agent as defined in Eq. (6), to make $J(C_t, A_t) + \int_0^t f(C_s, J(C_s, A_s))ds$ a martingale under the most pessimistic probability measure, we have

$$\mathfrak{D}^{C,A}J + f(C,J) - \theta^*(A)J_C C\sigma_c = 0.$$

Conjecture that J has the functional form in Eq. (12). Substituting it into the differential equation above, we obtain

$$(1-\gamma)\left(\mu_c - A - \frac{1}{2}\gamma\sigma_c^2 + \left(\frac{L_A}{L}\theta\sigma_a - \frac{1}{2}\gamma\sigma_c\sigma_{ca}\right)\sigma_c\sigma_{ca}A\right) + \frac{\mathfrak{D}^A L^\theta}{L^\theta} + \frac{\theta}{L} - \beta\theta = 0, \qquad \gamma, \psi \neq 1. \tag{B.1}$$

Eq. (11) in Proposition 1 is a special case with the correlation parameter σ_{ca} equal to zero.

To solve the model, we employ Collin-Dufresne and Goldstein (2005)'s log-linear approximation that essentially minimizes the expected squared approximation error. Specifically, we seek an exponential affine expression for the price-consumption ratio as presented in Eq. (13) such that the left-hand side of Eq. (B.1) is linear in A_t except for the θ/L term. In that case, the parameters η_0 and η_1 can be obtained by solving the following optimization problem:

$$\min_{\eta_0,\eta_1} E\left[(n_0 + n_1 A_t - \theta e^{-\eta_0 - \eta_1 A_t})^2\right],$$

where

$$-n_0 = (1-\gamma)\left(\mu_c - \frac{1}{2}\gamma\sigma_c^2\right) + \theta(\eta_1\kappa\bar{A} - \beta),$$

$$-n_1 = (1-\gamma)\left(\left(\eta_1\theta\sigma_a - \frac{1}{2}\gamma\sigma_c\sigma_{ca}\right)\sigma_c\sigma_{ca} - 1\right) - \theta\left(\eta_1\kappa - \frac{1}{2}\theta\eta_1^2\sigma_a^2\right).$$

Fig. B.1 makes a comparison of this approximate price-consumption ratio with the one numerically solved from Eq. (11). These two solutions turn out to be fairly close to each other. Indeed, when the ambiguity level is around its long-run mean, they are almost indistinguishable from each other. This result demonstrates the accuracy of the exponential affine approximation.

With first-order conditions for the optimal consumption choice under ambiguity, Chen and Epstein (2002) drive the following SDF:

$$M_t = e^{\int_0^t f_J(C_s,J_s)ds} f_C(C_t, J_t) Z_t^{\theta*},$$

where $Z^{\theta*}$ is the Radon–Nikodym derivative corresponding to the worse-case prior. The multiple-priors preference is reflected in the presence of $Z^{\theta*}$: if P_0 is the only belief in the prior set $\mathscr{P}$, it degenerates to one, and the SDF becomes the standard one in Duffie and Epstein (1992) and Duffie and Skiadas (1994).

To prove Proposition 2, we apply Itô's lemma to the pricing kernel. It follows that

$$r_t = -E\left(\frac{dM_t}{M_t dt}\right) = \gamma\left[\mu_c - A - \frac{1}{2}(1+\gamma)(1+\sigma_{ca}^2 A)\sigma_c^2 + \frac{L_A}{L}(\theta-1)\sigma_c\sigma_a\sigma_{ca}\right] - \frac{\mathfrak{D}^A L^{\theta-1}}{L^{\theta-1}} - \frac{\theta-1}{L} + \beta\theta.$$

Connecting it with Eq. (11), we obtain the risk-free rate as a linear function of the ambiguity level

$$r_t = \varrho_0 + \varrho_1 A_t,$$

where

$$\varrho_0 = \mu_c/\psi + \beta - \frac{1}{2}\gamma(1+1/\psi)\sigma_c^2, \tag{B.2a}$$

$$\varrho_1 = -1/\psi + \frac{1}{2}(\theta-1)\eta_1^2\sigma_a^2 + \left((\theta-1)\eta_1\sigma_a - \frac{1}{2}\gamma(1+1/\psi)\sigma_c\sigma_{ca}\right)\sigma_c\sigma_{ca}. \tag{B.2b}$$

If consumption growth is a random walk independent of innovations in the ambiguity level, the last term in Eq. (B.2b) vanishes.

It follows that the dual role time-varying ambiguity serves in equilibrium asset pricing is also reflected in its impact on the risk-free rate. The first term on the right-hand side of Eq. (B.2b) captures the effect of ambiguity on intertemporal smoothing. Also persisting in the case of constant ambiguity, it underlines the fact that ambiguity about the distribution of future payoffs is a first-order concern. However, the second term is absent in models with either time-invariant ambiguity or constant relative risk aversion ($\theta = 1$). The agent's preference for early resolution of uncertainty would lower the equilibrium interest rate because she wants to divest from risky assets when the future level of ambiguity becomes highly uncertain. Therefore, both effects enhance the agent's saving motive and thus help to explain the risk-free rate puzzle.

As shown in Table 6, the drift perturbation term in Eq. (B.2b) decreases the mean of r_t by merely $\bar{A}/\psi = 0.61\%$, which is not enough to account for the risk-free rate puzzle. Therefore, the second term $(\theta-1)\eta_1^2\sigma_a^2/2$, which is missing in the constant-ambiguity case, is necessary to capture the low mean of historical interest rates. It symbolizes the precautionary saving motive caused by unknown future levels of ambiguity and leads to another 0.48% reduction in the unconditional expectation of r_t.[60] Hence,

[60] Note that allowing for serial covariations between c_t and A_t results in another 0.05% decrease in ϱ_1, which partially accounts for the gap between risk-free rates generated by the models with and without time-varying ambiguity. This cross-shock effect is reflected by the last term in Eq. (B.2b).

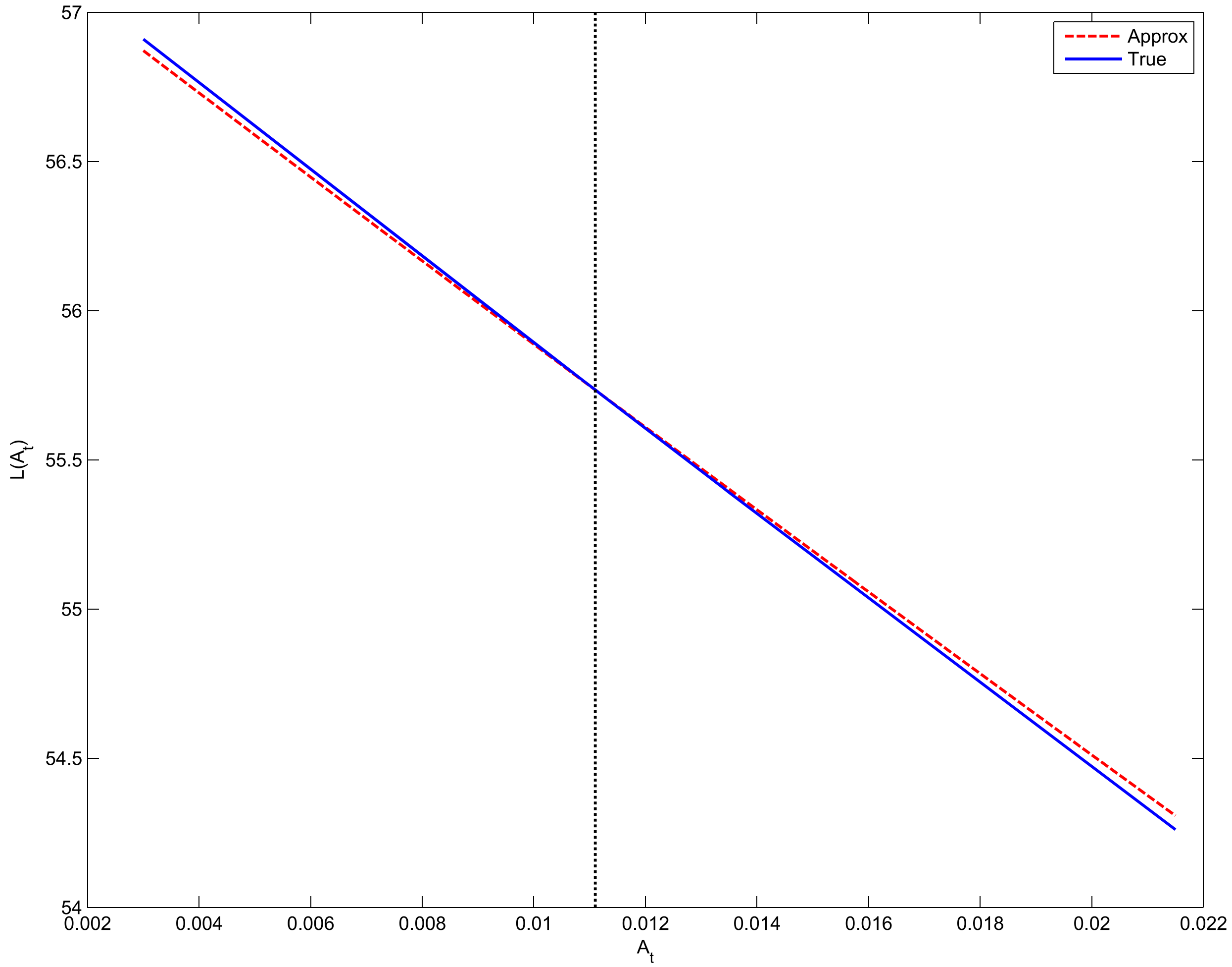

Fig. B1. Approximate solution for the price-consumption ratio.
This figure shows the accuracy of our approximation of the price-consumption ratio L_t as an exponential affine function of the ambiguity level A_t. The dashed red line displays the approximate solution for $L(A)$ and the solid blue line the solution literally evaluated from the differential Eq. (11). The ODE is solved using the MATLAB function "ode45" with the initial condition $L(0) = \frac{\theta}{(1-\gamma)(\mu_c - \frac{1}{2}\gamma\sigma_c^2) - \beta\theta}$. The grid line to the x-axis represents the historical mean of A_t. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

time-varying ambiguity is crucial for the model to match the key statistics of risk-free rates.

Appendix C. A PDE-based characterization of equity and bond pricing

As a standard approach to endogenous default problems, we look for a model solution analogous to that in Leland (1994a,b), with the additional spatial variable A_t. This type of free-boundary problem is usually solved by verifying that a given candidate function e actually coincides with E and a corresponding stopping time $\tau_{\mathcal{B}}$ is optimal. The following proposition is derived from the variational inequality verification theorem proved in Øksendal (2003).

Proposition 3. Suppose we can find a function $e : \bar{\mathcal{S}} \to \mathbb{R}$ *such that* $e \in \mathcal{C}^1(\mathcal{S}) \bigcap \mathcal{C}(\bar{\mathcal{S}})$ *and* $e \geq 0$ *on* $\mathcal{S}$*. Define*

$$\mathcal{B} = \{(\delta_t, A_t) \in \mathcal{S}; e(\delta_t, A_t) > 0\}$$

as the continuation region.

(a) *Suppose* $\partial\mathcal{B}$ *is a Lipschitz surface. If*
   (i) (δ_t, A_t) *spends zero time at* $\partial\mathcal{B}$*, i.e.,* $E_t^Q\left[\int_t^{\tau_S} \mathcal{X}_{\partial D}(\delta_t, A_t) ds\right] = 0$,
   (ii) $e \in C^2(\mathcal{S}\backslash\partial\mathcal{B})$ *with locally bounded derivatives near* $\partial\mathcal{B}$,
   (iii) $\mathfrak{D}e(\delta_t, A_t) + \delta_e - r(A_t)e(\delta_t, A_t) \leq 0$ *on* $\mathcal{S}\backslash\partial\mathcal{B}$,
   then $e(\delta_t, A_t) \geq E(\delta_t, A_t)$ *for all* $(\delta_t, A_t) \in \mathcal{S}$.

(b) *Moreover, assume*
   (iv) $\mathfrak{D}e(\delta_t, A_t) + \delta_e - r(A_t)e(\delta_t, A_t) = 0$ *on* $\mathcal{B}$,
   (v) $\tau_{\mathcal{B}} = \inf\{t > 0; (\delta_t, A_t) \notin \mathcal{D}\} < \infty$ *for all* $(\delta_t, A_t) \in \mathcal{S}$,
   (vi) *the family* $\{e(\delta_\tau, A_\tau); \tau \in \mathcal{T}, \tau \leq \tau_{\mathcal{B}}\}$ *is uniformly integrable for all* $(\delta_t, A_t) \in \mathcal{S}$.
   Then $e(\delta_\tau, A_\tau) = E(\delta_\tau, A_\tau)$ *and* $\tau^* = \tau_{\mathcal{B}}$ *is an optimal default time.*

(c) *Define* $\mathcal{U} = \{(\delta_t, A_t) \in \mathcal{S}; \delta_{e,t} > 0\}$*. Suppose that for all* $(\delta_t, A_t) \in \mathcal{U}$ *there exists a neighborhood* $\mathcal{W}$ *of* (δ_t, A_t) *such that* $\tau_{\mathcal{W}} = \inf\{t > 0; (\delta_t, A_t) \notin \mathcal{W}\} < \infty$*. Then* $\mathcal{U} \subset$

$\{(\delta_t, A_t) \in \mathcal{S}; e(\delta_t, A_t) > 0\} = \mathcal{B}$. *Therefore it is never optimal to default while* $(\delta_t, A_t) \in \mathcal{U}$.

These results are deduced from the "high contact principle" (Øksendal, 1990; Brekke and Øksendal, 1991), which has been fruitfully applied in optimal stopping and stochastic waves. To find a reasonable guess for the continuation region $\mathcal{B}$, we refer to Proposition 3(c). In view of the expression for $\delta_{e,t}$, which is a nondecreasing function of δ_t, we conjecture that $\mathcal{B}$ has the form $\mathcal{B} = \{(\delta_t, A_t); \delta_t > \delta^*(A_t)\}$ such that $\mathcal{U} \subseteq \mathcal{B}$.[61] In other words, inspired by Goldstein et al. (2001) and Strebulaev (2007), we define the default time in terms of corporate earnings. Therefore, the optimal strategy of shareholders is to find out the critical default barrier δ^* that depends on the current level of ambiguity.

Proposition 3 characterizes the maximized equity value by the PDE listed in *(iv)*. The relevant boundary conditions derive from its $\mathcal{C}^1$-property. The continuity condition implies that

$$E(\delta^*(A), A) = 0. \tag{C.1}$$

Moreover, to ensure differentiability,[62] it is further required that

$$\lim_{\delta\to\delta^*} E_\delta = \lim_{\delta\to\delta^*} E_A = 0. \tag{C.2}$$

The market value of newly issued debt D, on the other hand, satisfies the PDE

$$\mathfrak{D}D(\delta, A) + (1-\tau_i)mC + m^2F - (r(A)+m)D(\delta, A) = 0,$$

$$\lim_{\delta\to\delta^*} D(\delta, A) = (1-\phi)mU(\delta^*(A), A), \tag{C.3}$$

$$\lim_{\delta\to\infty} D(\delta, A) = ((1-\tau_i)mC + m^2F)\mathscr{PV}^{r+m}(A), \tag{C.4}$$

$$\lim_{\delta\to\infty} D_\delta(\delta, A) = 0, \tag{C.5}$$

where $\mathscr{PV}$ is the expected present value operator

$$\mathscr{PV}^q(x_t) = E_t^Q\left[\int_t^\infty e^{-\int_t^s (q(x_u)du}) ds\right].$$

The upper boundary conditions (C.4) and (C.5) imply that when the firm's payoff grows to infinity, the possibility of default becomes meaningless so that the debt value tends toward the price of a default-free claim to the continuous cash flow $(1-\tau_i)mC + m^2F$. The expression $U(\delta_t, A_t)$ in Eq. (C.3) denotes the value of unlevered assets that can be written as

$$U(\delta_t, A_t) = \delta_t L^o(A_t),$$

where $L^o(A_t)$ is the price-earnings ratio. Given the affine model structure, both $\mathscr{PV}^r(A)$ and $L^o(A_t)$ would be solved with a log-linear approximation:

$$L^o(A) \approx e^{\xi_0+\xi_1 A}, \tag{C.6}$$

$$\mathscr{PV}^r(A) \approx e^{\zeta_0+\zeta_1 A}. \tag{C.7}$$

Solutions to ξ_0 and ξ_1 can be derived in the same manner as in the case of the price-consumption ratio L_t. By Itô's product rule, the expected market return on a claim to aggregate output can be written as

$$r - \frac{1}{L^o} = \frac{1}{dt}E^Q\left[\frac{dL^o}{L^o} + \frac{dO}{O} + \frac{dL^o}{L^o}\frac{dO}{O}\right].$$

Eq. (C.6) indicates that we approximate the $L^o(A)$ as a simple exponential formula with ξ_0 and ξ_1 minimizing the mean squared error

$$\min_{\zeta_0,\zeta_1} E\left[(m_0 + m_1 A_t - e^{-\xi_0-\xi_1 A_t})^2\right],$$

where

$$-m_0 = -\varrho_0 + \mu_c - \gamma\sigma_c\sigma_o\sigma_{oc} + \xi_1\kappa\bar{A},$$

$$-m_1 = -\varrho_1 - \sigma_o\sigma_{oc}/\sigma_c - \xi_1\kappa - \left((1-\theta)\eta_1\xi_1 - \frac{1}{2}\eta_1^2\right)\sigma_a^2 + ((1-\theta)\eta_1 - \xi_1)\sigma_o\sigma_a\sigma_{oa} - (\xi_1\sigma_a + \sigma_o\sigma_{oa})\gamma\sigma_c\sigma_{ca}.$$

Likewise, parameters ζ_0 and ζ_1 in Eq. (C.7) can be identified by solving the following optimization problem:

$$\min_{\zeta_0,\zeta_1} E\left[(l_0 + l_1 A_t - e^{-\zeta_0-\zeta_1 A_t})^2\right],$$

where

$$-l_0 = -\varrho_0 + \zeta_1\kappa\bar{A},$$

$$-l_1 = -\varrho_1 - \zeta_1\kappa + \left(\frac{1}{2}\zeta_1^2 - (1-\theta)\eta_1\zeta_1\right)\sigma_a^2 - \gamma\zeta_1\sigma_c\sigma_a\sigma_{ca}.$$

Appendix D. Dynamic properties of asset prices

In this section, we review some stylized facts about the dynamics of equity returns and extend them to the corporate debt market. Section D.1 investigates the forecasting power of the price-dividend ratio: using this ratio, equity returns and credit spreads are predictable, but consumption and dividend growth are not. The weak correlation between asset markets and macroeconomics, as discussed in Section D.2, poses another serious challenge. We find that our model can reproduce these empirical regularities.

D1. Predictive regressions

Panel A in Table D1 presents long-horizon regressions of excess stock returns on the log price-dividend ratio in simulated and historical data. Since our sample spans only a 26-year period, the predictive regressions are run at a quarterly frequency. Mindful of issues arising from the use of overlapping observations, we compute Hodrick (1992) standard errors to remove the moving average structure in the error terms. Like Campbell and Shiller (1988) and Fama and French (1988), we find that high equity prices imply low expected returns;[63] both the absolute values of slope coefficients and the R^2 increase with the return horizon.

[61] This conjecture is easily verified using the results established in Mordecki (2002) for a general Lévy process.

[62] While the continuous pasting condition (C.1) is necessary, the smooth pasting condition (C.2) need not hold in general, although it holds in the optimal default problem. See Alili and Kyprianou (2005) for a detailed discussion.

[63] Based on Hodrick (1992)'s t-statistics, the predictive ability of the price-dividend ratio is marginally significant at the horizon of one year,

Table D1
Long-horizon predictive regressions.
This table shows regression results of excess stock returns $\sum_{j=1}^{4h}(r_{e,t+j/4} - r_{t+j/4})$, credit spreads $CS_{Baa,t+h}$, consumption growth $\sum_{j=1}^{4h}\Delta c_{t+j/4}$, and dividend growth $\sum_{j=1}^{4h}\Delta\delta_{e,t+j/4}$ on the log price-dividend ratios for $h = 1, 3$, and 5 years. In Panels A, C, and D, "s.e." denotes Hodrick (1992) standard errors; in Panel B it corresponds to Newey and West (1987) standard errors. The entries for the model are the mean, 5%, and 95% percentiles (in brackets) based on 1,000 simulated samples with 26×252 daily observations that are aggregated to a quarterly frequency. The sample period spans from 1985 to 2010.

Panel A: Predictability of excess returns

Horizon	Data			Unlevered		Levered	
	Coeff.	s.e.	R^2	Coeff.	R^2	Coeff.	R^2
1	−0.15	(0.08)	0.10	−0.35	0.19	−0.18	0.13
3	−0.39	(0.23)	0.16	−0.73	0.33	−0.49	0.21
5	−0.78	(0.44)	0.29	−1.08	0.43	−0.78	0.30

Panel B: Predictability of credit spreads

Horizon	Data			Individual		Portfolio	
	Coeff.	s.e.	R^2	Coeff.	R^2	Coeff.	R^2
1	0.45	(0.24)	0.04	0.39	0.03	0.42	0.06
3	0.81	(0.26)	0.14	0.56	0.06	0.69	0.21
5	0.84	(0.38)	0.15	0.58	0.07	0.64	0.17

Panel C: Predictability of dividend growth

Horizon	Data			Unlevered		Levered	
	Coeff.	s.e.	R^2	Coeff.	R^2	Coeff.	R^2
1	0.0172	(0.0477)	0.0036	−0.0105	0.03	−0.0068	0.01
				[−0.07, 0.05]	[0.00, 0.09]	[−0.04, 0.04]	[0.00, 0.05]
3	0.0237	(0.1261)	0.0027	−0.0258	0.07	−0.0092	0.02
				[−0.17, 0.12]	[0.00, 0.20]	[−0.14, 0.13]	[0.00, 0.11]
5	0.1751	(0.2487)	0.1244	−0.0378	0.10	−0.0151	0.04
				[−0.25, 0.17]	[0.00, 0.26]	[−0.20, 0.18]	[0.00, 0.14]

Panel D: Predictability of consumption growth

Horizon	Data			Unlevered		Levered	
	Coeff.	s.e.	R^2	Coeff.	R^2	Coeff.	R^2
1	0.0025	(0.0068)	0.0052	−0.0033	0.02	−0.0021	0.02
				[−0.04, 0.03]	[0.00, 0.08]	[−0.03, 0.03]	[0.00, 0.06]
3	−0.0045	(0.0166)	0.0035	−0.0094	0.05	−0.0026	0.02
				[−0.11, 0.09]	[0.00, 0.20]	[−0.04, 0.04]	[0.00, 0.09]
5	−0.0070	(0.0326)	0.0075	−0.0135	0.07	−0.0074	0.04
				[−0.17, 0.13]	[0.00, 0.27]	[−0.09, 0.08]	[0.00, 0.17]

The last four columns show that the model captures the negative relation between the equity premium and the price-dividend ratio: high equity prices relative to dividends imply low ambiguity levels and therefore predict low future expected returns on stocks in excess of the risk-free rate. For unlevered equity, the model-implied predictability appears too strong compared to that suggested by the data because the log P/D ratio and unlevered equity premium are perfectly correlated. Introducing default risk into the equity produces more realistic regression statistics. At the five-year horizon, the model-implied coefficient and R^2 are almost identical to those in the data.

but it disappears for longer horizons. We emphasize that our focus is not on testing the predictability of stock returns. Even if we ignore the issues associated with statistical inference for the long-horizon predictive regressions, existing evidence for the return predictability is sensitive to changing samples (Goyal and Welch, 2003; Ang and Bekaert, 2007). Instead, the question examined here is whether the model can reproduce the apparent forecasting power of the price-dividend ratio.

The price-dividend ratio's link with the ambiguity level also connects it to credit spreads. Given that ambiguity follows a mean-reverting process, we may expect some long-horizon predictability of credit spreads. This is confirmed by results in Panel B: over the long run, high price-dividend ratios forecast high credit spreads for the Baa bond index; at the one-year horizon, the P/D ratio is not a powerful predictor.[64] Regressions with the simulated data display a similar pattern. For an individual Baa firm, the model-generated R^2s are substantially lower than their empirical counterparts because of the presence of idiosyncratic volatility. If we consider only systematic variations in the corporate cash flow, the model-implied predictability is aligned with that observed from market data, as shown in the last two columns.

[64] Shorter horizon results, not reported in the table, suggest a negative relation, which is consistent with the persistence in the ambiguity evaluation. But the predictability is not significant either.

Table D2

Correlations of consumption growth with equity returns and credit spreads.

This table shows historical and model-implied correlations of consumption growth with stock returns, and with changes in credit spreads, for different leads and lags. The column labeled “Data” in Panel A reports the correlation between the growth rate of aggregate consumption and returns on the NYSE/AMEX/NASDAQ value-weighted index. The “Data” columns in Panel B display the correlations of consumption growth with changes in yield spreads of Barclays US corporate Baa bond index and Barclays US high-yield Ba bond index over Barclays US Treasury bond index. The entries for the model are the means, 90% confidence intervals based on 1000 simulated samples with 26×252 daily observations that are aggregated to a quarterly frequency. All empirical data involved cover the period from 1985 to 2010 and are sampled at a quarterly frequency.

Panel A: Consumption growth w/ stock returns

	Data	Model		
		Unlevered	Levered	90% intvl.
$r_{e,t}$, Δc_{t-2}	−0.05	−0.01	−0.01	[−0.21, 0.19]
$r_{e,t}$, Δc_{t-1}	−0.01	−0.01	−0.02	[−0.22, 0.18]
$r_{e,t}$, Δc_{t}	0.10	0.26	0.22	[0.03, 0.34]
$r_{e,t}$, Δc_{t+1}	0.15	0.00	0.00	[−0.19, 0.17]
$r_{e,t}$, Δc_{t+2}	0.13	0.00	0.01	[−0.18, 0.19]

Panel B: Consumption growth w/ changes in credit spreads

	Baa			Ba		
	Data	Model	90% intvl.	Data	Model	90% intvl.
ΔCS_t, Δc_{t-2}	0.17	0.02	[−0.16, 0.22]	0.19	0.01	[−0.17, 0.19]
ΔCS_t, Δc_{t-1}	0.15	0.02	[−0.17, 0.21]	0.12	0.00	[−0.18, 0.17]
ΔCS_t, Δc_{t}	−0.12	−0.20	[−0.33, −0.03]	−0.17	−0.25	[−0.42, −0.08]
ΔCS_t, Δc_{t+1}	−0.16	0.00	[−0.18, 0.18]	−0.18	−0.01	[−0.18, 0.17]
ΔCS_t, Δc_{t+2}	−0.21	−0.01	[−0.20, 0.18]	−0.14	−0.02	[−0.20, 0.16]

The historical variation in price-dividend ratios could also be driven by changes in expected dividend growth, in addition to movements in the equity premium. However, previous studies (Campbell, 1999; Lettau and Ludvigson, 2001) find that the P/D ratio is not useful for forecasting either consumption growth or dividend growth. In Panel C, we report the model-implied predictability in dividend growth; the estimated slope coefficients are positive but insignificant at any horizon. For both unlevered and levered equity, the average coefficients implied from the simulated samples are negative[65]; however, they are all within one standard error of corresponding empirical estimates. At the same time, the model-implied 90% confidence bands for regression coefficients and R^2 contain the data counterparts.

For comparative purposes, Panel D provides regression results where consumption growth is used as the dependent variable, as this regression specification is also estimated in Bansal and Yaron (2004) and Wachter (2013). Again, both the data and the model suggest that price-dividend ratios do not have any statistical or economic significance in predicting growth rates. The model-implied average slope coefficients match the data well. While the empirical R^2s are slightly lower than the model's predictions, they still fall within the 90% confidence band. Overall, the model accounts for the absence of predictability in dividend and consumption growth.

D.2. Correlations

Cochrane and Hansen (1992) and Campbell and Cochrane (1999) point out that the weak correlation between consumption growth and stock returns in the US data is inconsistent with the implications of many consumption-based models. For models solely driven by shocks to consumption growth, these two series should be highly, if not perfectly, correlated. Similarly, if we simply embed a Merton model inside these equilibrium models, we would expect changes in credit spreads to have a strong but negative correlation with consumption growth. But, empirically, we find that this correlation is as low as it is in the case of equity returns. These low correlations tend to foil economists' attempts to link the stock and credit markets with the macroeconomics. However, adding dynamics to the agent's ambiguity aversion allows for new insights to understand the lack of comovement in these time series.

Panel A in Table D2 shows the correlation of consumption growth with market returns in actual and simulated data. At a quarterly horizon, the contemporaneous correlation is measured at 0.10, close to the 0.12 reported in

[65] Note that the positive slope coefficient is not a permanent feature of predictive regressions of dividend growth. For example, negative, but insignificant, coefficients are documented by Campbell (1999), whose sample covers the 1947Q2-1996Q3 period. According to Eq. (15), there is zero predictability in the firm's cash flow growth. In our simulation exercise, the P/D ratio exhibits a weak negative correlation with the future growth rate in the long run due to the correlation term $\sigma_{oa}\sigma_a\sqrt{A_t}dB_{A,t}$. For unlevered equity, its P/D ratio is linear in the ambiguity level. So when it is currently high, the ambiguity level is likely to bounce back to its long-run mean in the future. This tendency will negatively affect output/dividend growth, especially in the long run. For levered equity, this negative correlation is further weakened by the nonlinearity in the relation between the P/D ratio and the ambiguity level.

Campbell and Cochrane (1999). To account for this weak correlation, exogenous shocks to the economy, besides consumption shocks, are necessary. With innovations in the ambiguity level, the model implies a mean correlation of 0.22 and a 90% confidence interval, which embrace the empirical correlation coefficients.

When studying consumption's correlation with credit spreads, we examine the spreads for both investment-grade and high-yield bonds. To disentangle variations in the credit components of yield spreads, Panel B focuses on the results of the lowest investment-grade rating, Baa, and the highest speculative-grade rating, Ba. In the data, there is a negative but weak correlation between spread changes and economic fundamentals. In our model, changes in credit spreads result only from uncertainty in ambiguity innovations, and as such they should be orthogonal to consumption growth without the correlation term. Incorporating the empirical comovement between the consumption and ambiguity processes leads to correlation coefficients of −0.20 and −0.25. The higher model-implied correlation for high-yield bonds (compared to investment-grade ones) is consistent with the data.

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